Iterative Geometry Calibration from Distance Estimates for Wireless Acoustic Sensor Networks

Tobias Gburrek, Joerg Schmalenstroer, Reinhold Haeb-Umbach
University of Paderborn, Germany

Motivation

- Task: Geometry calibration in wireless acoustic sensor networks from observed speech signals
- Estimate relative positions of nodes
- Observations: DNN-based distance estimates from signals Coherence-to-Diffuse-Power-Ratio (CDR) (see [1])
- Advantages of the approach:
  - Solely works with distance estimates between acoustic sources and sensor nodes
  - No special calibration signals required, natural speech sufficient.
  - Requires only coarse synchronization between sensor nodes

Optimization problem

- $N$ sensor nodes at positions $\Omega_P := \{P_1, \ldots, P_N\}$
- $K$ spatially distributed acoustic sources at positions $\Omega_O := \{O_1, \ldots, O_K\}$
- GARDE delivers estimates of all unknown positions $\Omega_P \cup \Omega_O$ by solving

\[
\hat{\Omega} = \arg \min_{\Omega} \sum_{k=1}^K \sum_{n=1}^N \left( \hat{d}_{k,n} - |P_n - O_k|^2 \right)^2.
\]

- Gradient of $J(\Omega)$ w.r.t. $g$-th sensor position $P_g$

\[
\hat{J}_{\Omega P_g} = 0 \iff \sum_{n=1}^N (P_g - O_n) \left( d_{g,n}^2 - |P_n - O_k|^2 \right) = 0
\]

- Gradient of $J(\Omega)$ w.r.t. $h$-th acoustic source position $O_h$

\[
\hat{J}_{\Omega O_h} = 0 \iff \sum_{n=1}^N (P_n - O_h) \left( d_{h,n}^2 - |P_n - O_h|^2 \right) = 0
\]

- No closed form solution $\rightarrow$ Iterative algorithm required
  - Assume either $\Omega_P$ or $\Omega_O$ to be known
  - Similarity of (2) and (3) enables use of common functions

Weighted Least Squares

\[
\begin{bmatrix}
2\hat{d}_{1,x}^2 & 2\hat{d}_{1,y}^2 \\
2\hat{d}_{2,x}^2 & 2\hat{d}_{2,y}^2 \\
\vdots & \vdots \\
2\hat{d}_{N,x}^2 & 2\hat{d}_{N,y}^2
\end{bmatrix}
\begin{bmatrix}
\hat{O}_{x,1} \\
\hat{O}_{y,1} \\
\vdots \\
\hat{O}_{y,N}
\end{bmatrix} =
\begin{bmatrix}
\hat{d}_{1,x}^2 + \hat{d}_{1,y}^2 - \hat{d}_{1,k}^2 \\
\hat{d}_{2,x}^2 + \hat{d}_{2,y}^2 - \hat{d}_{2,k}^2 \\
\vdots \\
\hat{d}_{N,x}^2 + \hat{d}_{N,y}^2 - \hat{d}_{N,k}^2
\end{bmatrix}
\]

- Estimate location $\hat{O}_k$ with weights $w_k = 1/\hat{d}_k^2$

\[
\hat{O}_k = \left( R^T W R \right)^{-1} R^T W b + P_o
\]

Multidimensional Scaling Initialization

- Approximation by triangular inequality

\[
\max(d_{ij} - d_{il}) \leq D_{ij} \leq \min(d_{il} + d_{lj})
\]

Experiments

- Comparison between CRLB of estimator and RMSE of observation and sensor positions. Number of iterations and annealing rounds were equally chosen.

Conclusions

- GARDE algorithm: Iterative WLS-based algorithm for geometry calibration of WASNs
  - Positions of sensor nodes and acoustic sources
- Derived CRLB for the geometry calibration approach

1Python implementation of GARDE is available in the paderwa paper repository: https://github.com/Epnt/paderwa