Single-Pixel Camera Sensing Matrix Design for Hierarchical Compressed Spectral Clustering

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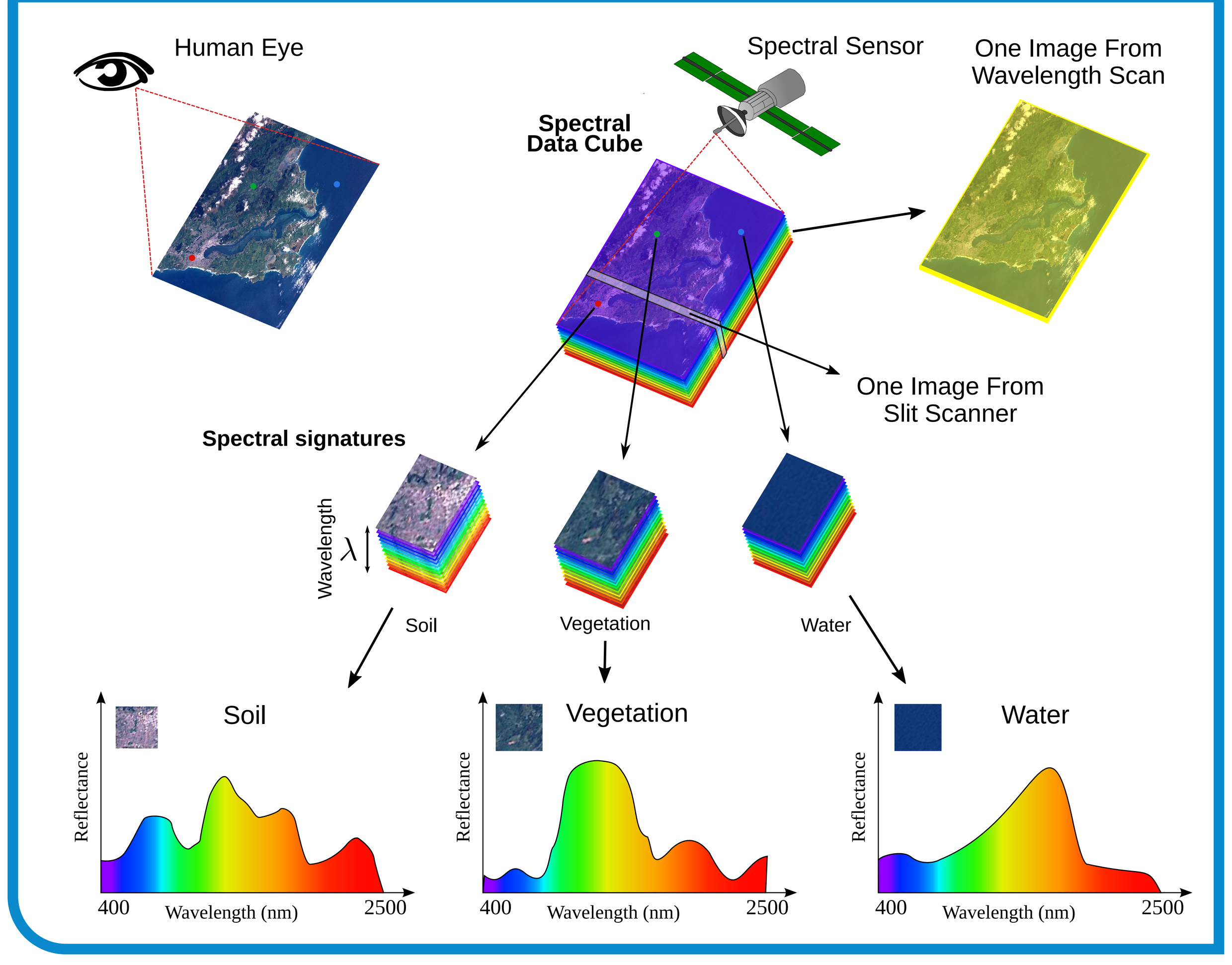
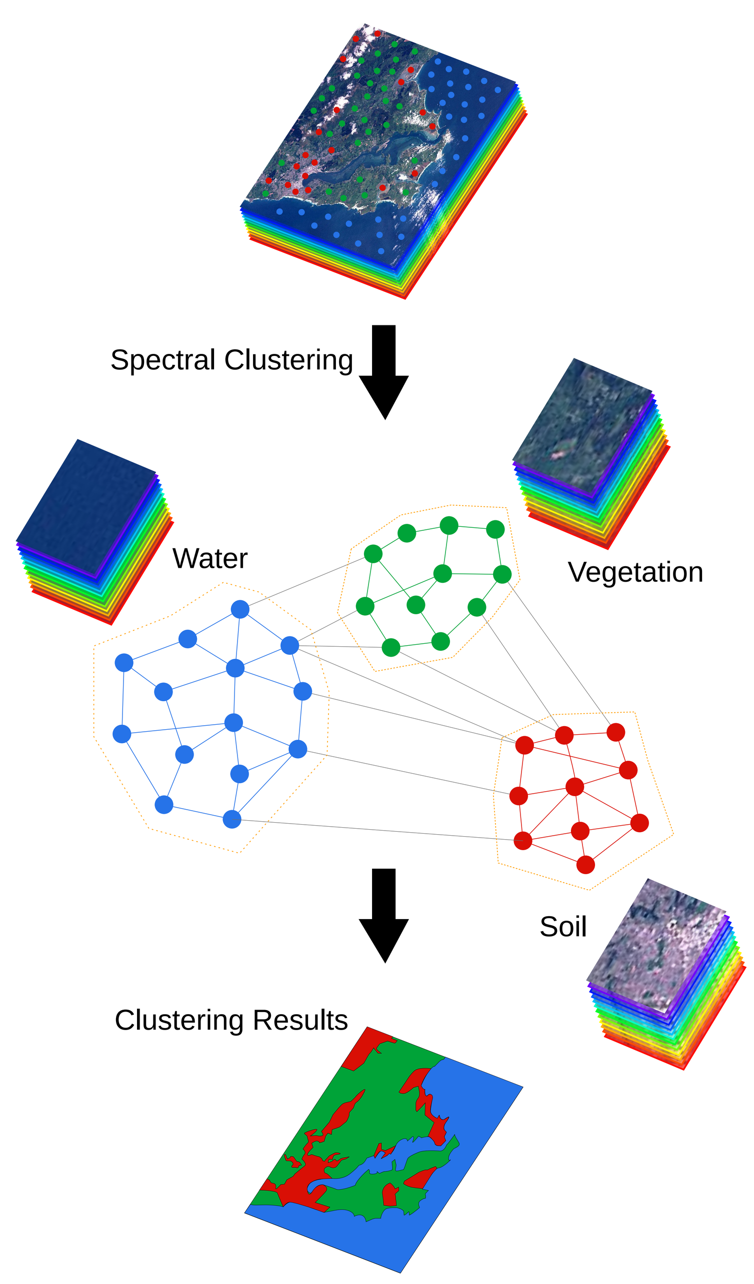
Spectral Imaging

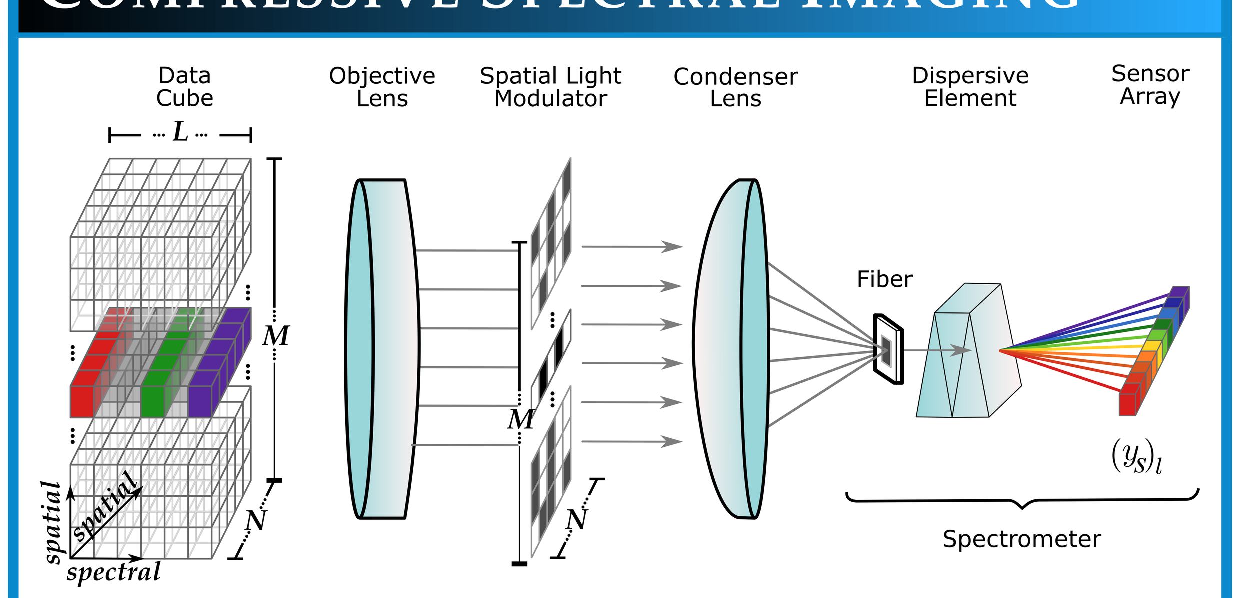


Spectral Clustering



- Given $\hat{N} = M \cdot N$ points, define a similarity matrix $\mathbf{A} \in \mathbb{R}^{\hat{N}\times\hat{N}}$ using ϵ-neighborhood graph.
- $\mathbf{L} = \mathbf{I} - \mathbf{D}^{-1/2}\mathbf{A}\mathbf{D}^{-1/2}$, where $D_{ii} = \sum_j A_{ii}$.
- Compute the first k eigenvectors $\mathbf{u}_1, \cdots, \mathbf{u}_k$ of $\mathbf{L}$ and define the matrix $\mathbf{U} \in \mathbb{R}^{\hat{N}\times k} = \left[\mathbf{u}_1^T, \cdots, \mathbf{u}_k^T\right]^T$
- Perform k-means on the rows of $\mathbf{U}$ and obtain the clusters $\mathbf{C}_1, \cdots, \mathbf{C}_k$.

Compressive Spectral Imaging



- Sense and simultaneously reduce the data-dimension.
- For the l-th spectral band, and using K shots, the acquisition scheme is expressed as

$$\mathbf{y}_l = \mathbf{H}\mathbf{f}_l, \tag{2}$$

where $\mathbf{H} = \left[\mathbf{h}_1^T, \cdots, \mathbf{h}_K^T\right]$, $\{\mathbf{h}_k\}_1^K$ is the vector form of the coding pattern used in the k-th shot.

- In general,

$$\mathbf{y} = \hat{\mathbf{H}}\mathbf{f}, \tag{3}$$

where $\hat{\mathbf{H}} = \mathbf{I}_L \otimes \mathbf{H}$, is a block diagonal matrix.

Proposed CSI Clustering

Taking into account the structure of Hadamard matrices, previous work in [1] proposes to design the sensing matrix for each band as

$$\mathbf{H} = \mathbf{W}\boldsymbol{\Delta}, \tag{1}$$

where $\mathbf{W} \in \{-1, 1\}^{K\times K}$ is a Hadamard matrix, and $\boldsymbol{\Delta} \in \mathbb{R}^{K\times MN}$ is a decimation matrix.

The proposed method consists of:

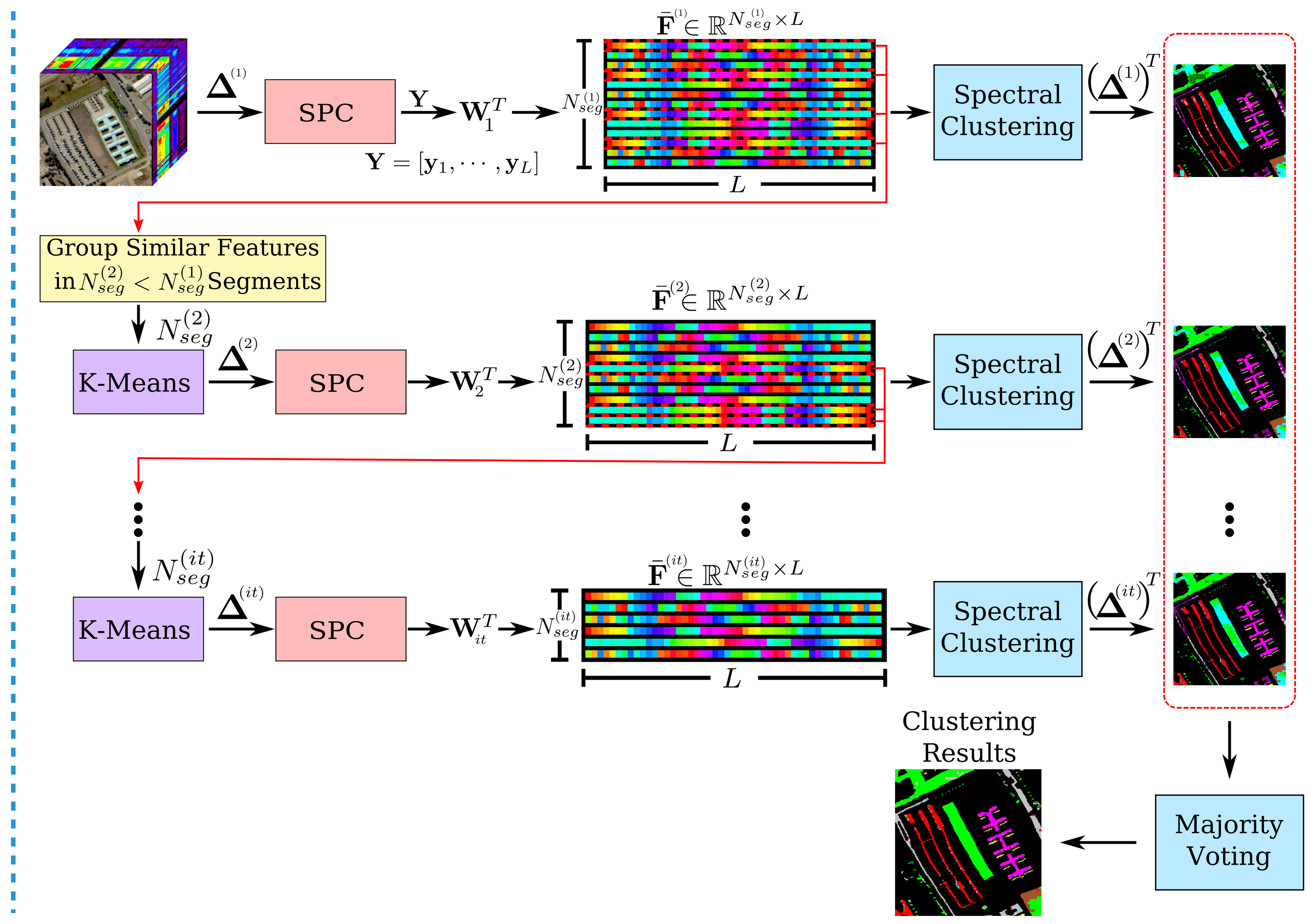
1. Design $\boldsymbol{\Delta}$ using k-means, see **Algorithm 1**. $\boldsymbol{\Delta}^{(1)}$ is designed uniformly.
2. Generate $\mathbf{W}_{it}$ and obtain the features from each band as $\bar{\mathbf{f}}_l = \mathbf{W}_{it}^T\mathbf{g}_l = \boldsymbol{\Delta}\mathbf{f}_l$.
3. Perform spectral clustering on rows of $\bar{\mathbf{F}} = \left[\bar{\mathbf{f}}_1, \cdots, \bar{\mathbf{f}}_L\right]$ and use $\boldsymbol{\Delta}$ to obtain the clustering results corresponding to the it scale. See **Algorithm 2**.

Algorithm 1 Downsampling Matrix Design

```
Input: N_seg, F̄
Output: Δ
procedure DSAMPLING_DESIGN(F̄, N_seg)
    k_idx ← k-means(F̄, N_seg)          ▷ k_idx contains the segment labels
    Δ ← zeros(N_seg, length(k_idx))
    for e ← 1 to N_seg do
        p^e ← find(k_idx = e)
        n_e ← length(p^e)
        for j ← 1 to n_e do
            (δ_e)^T_{(p^e)_j} = 1/n_e             ▷ Update each row of Δ
        end for
    end for
    return Δ
end procedure
```

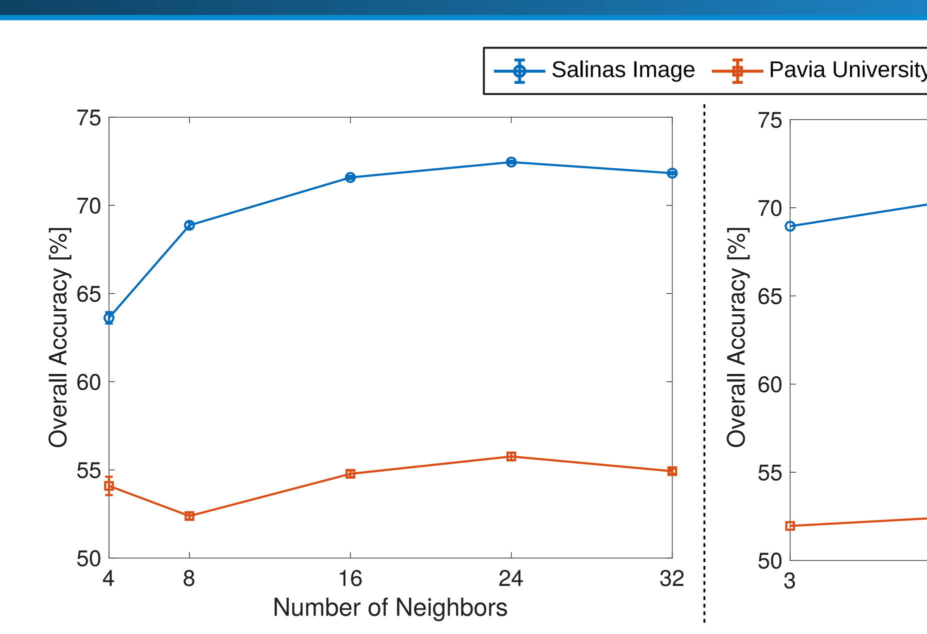
Algorithm 2 Data Clustering

```
Input: F̄ ∈ R^{N_seg×L}, Δ downsampling matrix, k clusters
Output: Segmentation of the spectral pixels: F_1, ···, F_k
procedure DATA_CLUSTERING(F̄, Δ, k)
    G ← Build_Sim_Graph(F̄)              ▷ k-nearest neighbor graph
    ◁ Obtain Cluster indices
    C̄_idx ← Spectral_Clustering(G, k)   ▷ Spectral Clustering [19]
    C_idx ← Δ^T C̄_idx                   ▷ Upsampling
end procedure
```

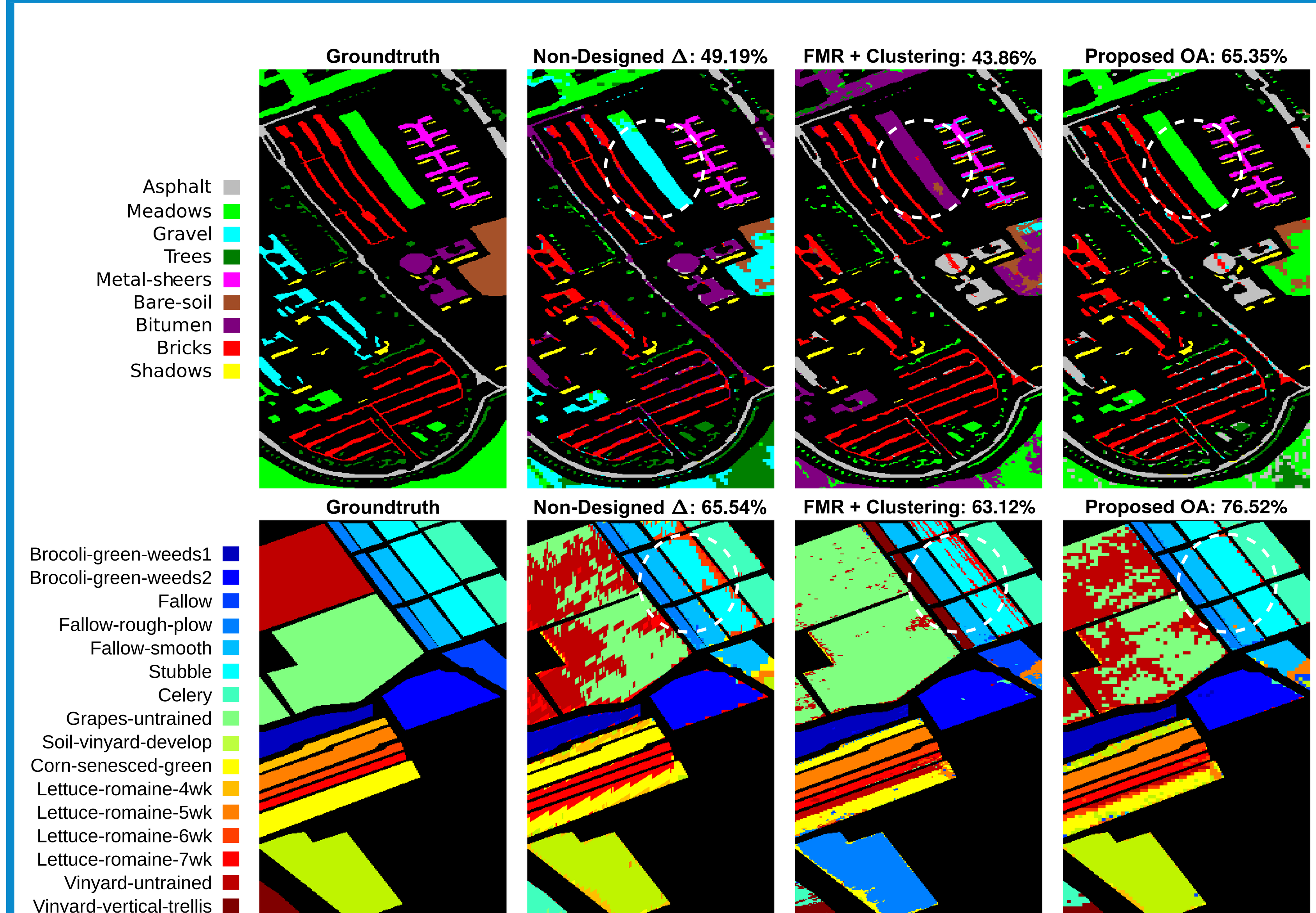


Parameters Analysis

The first two experiments were performed to show the sensitivity of the main parameters of the proposed method. Figure (a) and (b) show the overall accuracy as a function of the number of neighbors and scales in the proposed method, respectively.



Visual And Quantitative Results



Class	Non-Designed Δ	FMR+Clustering	Proposed
Broccoli-green-1	98.11	98.11	98.11
Broccoli-green-2	98.34	**98.77**	97.67
Fallow	7.91	**74.62**	8.70
Fallow-rough-plow	**96.48**	0.00	95.91
Fallow-smooth	92.91	**96.27**	93.09
Stubble	90.05	77.14	**97.07**
Celery	47.20	**99.17**	98.07
Grapes-untrained	78.91	**93.95**	55.15
Soil-vineyard-develop	93.42	0.48	**95.74**
Corn-senesced-green-weeds	**65.34**	59.43	50.27
Lettuce-romaine-4wk	19.19	2.15	**28.84**
Lettuce-romaine-5wk	0.00	**100.00**	99.53
Lettuce-romaine-6wk	0.00	**97.71**	96.51
Lettuce-romaine-7wk	0.00	83.27	**89.07**
Vineyard-untrained	0.00	3.10	**67.94**
Vineyard-vertical-trellis	**90.59**	82.79	77.37
AA	54.90	66.68	**78.07**
OA	62.55	63.12	**76.52**
Kappa	0.58	0.59	**0.74**

Table 1. Quantitative Results of different design approaches of $\boldsymbol{\Delta}$ for the Salinas Valley Image

Class	Non-Designed Δ	FMR+Clustering	Proposed
Asphalt	61.29	**91.75**	90.21
Meadows	26.30	21.98	**81.44**
Gravel	0.86	0.00	**4.05**
Trees	**77.84**	7.17	41.87
Metal sheets	**87.43**	71.23	86.77
Bare soil	30.11	**34.08**	32.63
Bitumen	**92.03**	0.00	0.00
Bricks	79.63	**93.97**	84.03
Shadows	89.14	**98.57**	92.12
AA	**60.51**	46.53	57.01
OA	49.19	43.86	**65.35**
Kappa	0.42	0.36	**0.57**

Table 2. Quantitative Results of different design approaches of $\boldsymbol{\Delta}$ for the Pavia University Image

References

[1] A. C. Sankaranarayanan, L. Xu, C. Studer, Y. Li, K. F. Kelly, & R. G. Baraniuk. Video Compressive Sensing for Spatial Multiplexing Cameras Using Motion-flow Models. In *SIAM Journal on Imaging Sciences* (2015).