

# Transmitted Beampattern Synthesis with Waveform Diverse Arrays Based on Arctangent Function

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## Abstract

In this paper, we apply a nonlinear frequency modulation scheme based on arctangent function to waveform diverse array system to design a transmitted beampattern to focus the signal power at a specific position. Distinct from most of the existing methodologies, the proposed method overcomes the problem that the focusing performance of the signal power will decrease when considering the propagation process of the transmitted signal. As a result, the transmitted signal power can be focused at a desired position and be lasting for a period of time. Numerical simulations validate the theoretical analysis of the proposed methodology.

## Introduction

On account of the ability to provide a range-angle dependent beampattern [1], frequency diverse arrays (FDA) technique offers potential technical support for wireless power transmission system, radar-based security system or some other application-specific system. Therefore, some researchers dedicated to using FDA technology to design transmitted beam patterns to focus signal power at a specific position. However, it is difficult to keep the focusing position unchanged after the signal power has been focused.

Initially, the research on FDA technology focused on the study of linear frequency offset. Some methodologies based on linear frequency offset has been proposed in [2]-[6]. The transmitted signals are required to be only coherently added at a specific position. However, the transmitted beampattern has multiple-maxima because of the unsuppressed angle dimension coupling. An FDA system consists of multiple sub-arrays with optimized transmit weight matrix has been proposed in [7] to design a range-angle dependent beampattern. However, when the desired focusing position has been changed, the transmit weight matrix need to be re-optimized. Some nonlinear offset (e.g., logarithmic frequency offset, random frequency offset) based techniques have been applied in [8]-[15], to design a beampattern with a single maximum. However, the focusing position will locate at the desired position only at instant . That is because these methodologies neglected the propagation process of transmitted signals. More specifically, the time variable  $t$  is considered to be a constant value in these methodologies. An FDA system consists of multiple sub-signals with complex weight coefficients optimized by convex optimization algorithm has been proposed in [16]. However, the complex weight coefficients of each sub-signal need to be optimized at each sampling point.

Different kinds of nonlinear frequency offset combined with nonlinear frequency modulation (NLFM) have been applied in [17]-[23] to focus the signal power at a specific position within the pulse duration. However, instead of fixed as a constant value, time variable  $t$  is defined as varying from 0 to  $T$ , where  $T$  is the pulse width of the transmitted signals. Therefore, it does not conform to the practical situation, and this constraint gives rise to certain limitations and consequently degrades the performance of FDA radar. Hence, the time variable  $t$  should represent the time that signals travel in the simulation space, practically. Using nonlinear frequency offset is a useful methodology to make the beampattern angle-dependent. And using nonlinear frequency modulation is a potential technique to make the beampattern range-dependent. Therefore, a possible technology to achieve a range-angle-dependent transmitted beampattern is to design a new nonlinear frequency modulation methodology and combine it with nonlinear frequency offset.

In this paper, we propose a nonlinear frequency modulation scheme based on arctangent function with the consideration of the propagation process of the transmitted signal. Different from those existing FDA radar systems, the phase of each signal in the proposed scheme is time variant. Therefore, the term "waveform diverse array" is more suitable than "frequency diverse array" to describe the proposed radar system. The proposed method avoids the inefficiencies in those existing methodologies.

## Authors Biographies

 Zhonghan Wang was born in Nantong, China, in 1993. He received the B.E. degree in telecommunication engineering from Xidian University in 2016. He is enrolled in Nanjing University of Science and Technology, Nanjing, Jiangsu Province, China as a Ph.D. Student. His main research interests are beamforming, radar signal processing, radar imaging and wireless communication.

 Yaoliang Song was born in Wuxi, China, on June 30, 1960. He received the B.Eng, the M.Eng, and the Ph.D degrees in Electrical Engineering from Nanjing University of Science and Technology, China, in 1983, 1986, and 2000 respectively. From Sept. 2004 to Sept 2005, he was a Researcher Fellow with the Department of Engineering Science at the University of Oxford. He is currently a Professor at Nanjing University of Science and Technology, and is heading the UWB Radar Imaging Group. His research interests include UWB communication, UWB Radar Imaging, and advanced signal processing.

 Tong mu received the B.E. degree in electronic information engineering from the Nanjing University of Science and Technology in 2014. He is now pursuing the Ph.D. degree in information and communication engineering. His main research interests are antenna array, radar signal processing and microwave imaging.

## Radar array structure

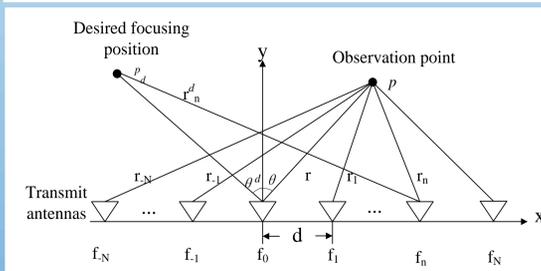


Figure 1. A uniformly-spaced linear array with  $2N+1$  elements.

## Transmitted signal model

$$S(t) = \sum_{n=-N}^N s_n(t) = \sum_{n=-N}^N w_n^* e^{-j2\pi[f_0 t + \psi_n(t + \Delta\tau_n)]}$$

$$w_n = e^{j2\pi f_0 \Delta\tau_n}$$

$$\psi_n(t) = \Delta f_n \cdot \left(t - \frac{T}{2}\right) \cdot \arctan\left[\frac{\alpha}{T} \cdot \left(t - \frac{T}{2}\right)\right] - \Delta f_n \cdot \frac{T}{2\alpha} \cdot \ln\left[\frac{\alpha^2}{T^2} \cdot \left(t - \frac{T}{2}\right)^2 + 1\right]$$

$$\Delta\tau_n = (r_n^d - r_n)/c$$

$$\Delta f_n = \frac{B}{\pi} \cdot \sin(|n|)$$

The resulting array factor observed at  $p(r, \theta)$  can be derived as

$$AF(t; r, \theta) = e^{-j2\pi f_0 t} \cdot \sum_{n=-N}^N w_n^* \cdot e^{j2\pi f_0 r_n/c - j2\pi \psi_n(t - r_n/c + \Delta\tau_n)}$$

The resulting power pattern at the desired position  $(r^d, \theta^d)$  can be expressed as

$$P(t; r^d, \theta^d) = |AF(t; r^d, \theta^d)|^2 = \left| \sum_{n=-N}^N e^{-j2\pi \psi_n(t - r_n^d/c + \Delta\tau_n)} \right|^2$$

## Simulations

The simulation results of the proposed methodology are reported and discussed in detail. The array system parameters are set to:  $N = 16$ ,  $f_0 = 6\text{GHz}$ ,  $B = 400\text{MHz}$ ,  $d = \lambda_0$ ,  $T = 10\text{ns}$  and  $\alpha = 30$ . The numerical simulation parameters are set to:  $r_{step} = 0.5\text{m}$  and  $\theta_{step} = \pi/360$  are the accuracies of space grid, the whole simulation region is  $\Omega = \{(r, \theta) | 0 < r < 12\text{m}, -\pi/2 < \theta < \pi/2\}$ , and the sampling frequency is  $f_s = 4f_0$ . Besides, the simulation time belongs to  $[0, 50\text{ns}]$ , where  $t = 0$  means the instant that the center antenna starts transmitting signals. And the desired focusing position is  $(3\text{m}, \pi/6)$ .

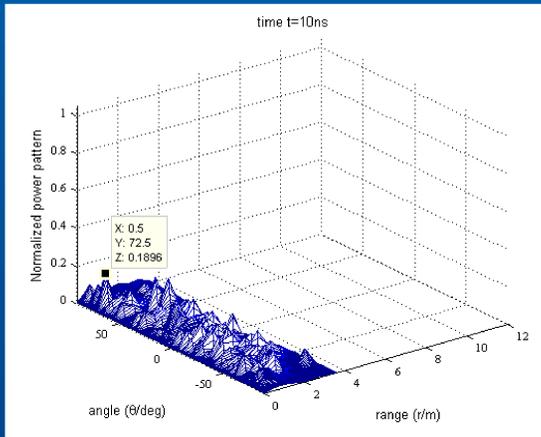


Figure 2. Normalized power pattern at  $t = 10\text{ns}$ .

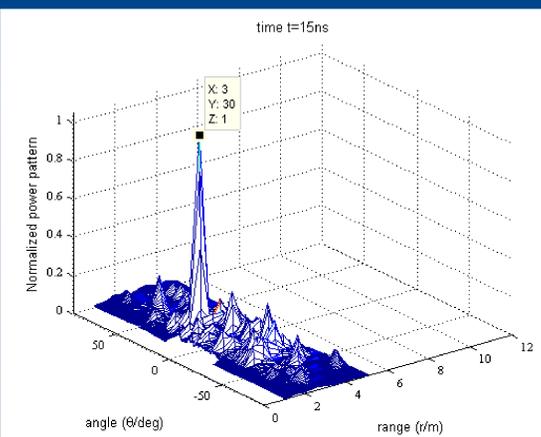


Figure 3. Normalized power pattern at  $t = 15\text{ns}$ .

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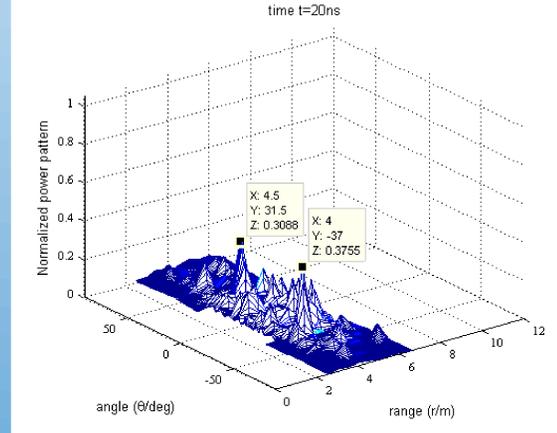


Figure 4. Normalized power pattern at  $t = 20\text{ns}$ .

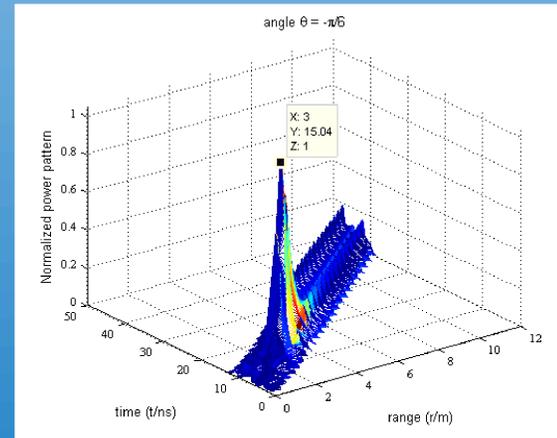


Figure 5. Normalized power pattern on the range-time dimensions at  $\theta = 30^\circ$

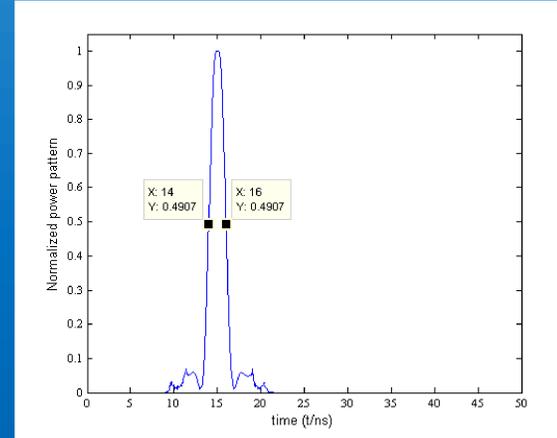


Figure 6. Normalized power pattern time curve of the desired position.

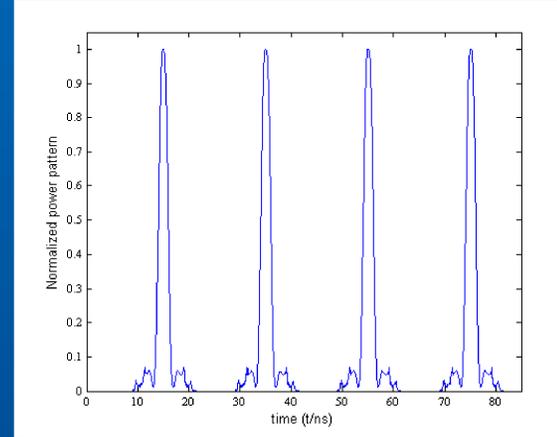


Figure 7. The received pulse signal at point  $(3\text{m}, \pi/6)$ .

## Conclusion

In this paper, a new transmitted beampattern synthesis method for waveform diverse array system radar has been proposed to focus the transmitted signal power at a specific position. The new methodology incorporates a nonlinear frequency modulation scheme based on arc-tangent function with non-uniform frequency offset. More specifically, the non-uniform frequency offset is suitable to make the transmitted beampattern angle-dependent; and the nonlinear frequency modulation based on arc-tangent function is appropriate to make the beampattern range-dependent. Different from those existing schemes, due to the arc-tangent function model, the frequency of each signal varies from greater than the carrier frequency to less than the carrier frequency. The closer the signal frequency is to the carrier frequency, the faster the signal frequency changes. Therefore, this feature makes the transmitted signal decoherent at any other locations except the desired position. Besides, this paper reports an inappropriate approximation applied in [17]-[23]. More specifically, propagation process of the transmitted signal has been ignored when deriving the array factor. Hence, it makes the focusing performance of the transmitted signal power degradative. And the proposed scheme has considered the propagation process of the transmitted signal. As a result, the proposed scheme can focus the signal power at the desired position for a period of time. Furthermore, the proposed array radar system is suitable to transmit a series of pulse signal because all the elements start to transmitting signal at the same time. Therefore, a pulse power signal with the same pulse repetition period will be observed at the desired position. This feature offers potential technical support for wireless power transmission system, radar-based security system or some other application-specific system.