SPARSE BOUNDED COMPONENT ANALYSIS FOR CONVOLUTIVE MIXTURES

Eren Babatas & Alper T. Erdogan

Koc University, Istanbul/Turkey

Contact Information:
Engineering Faculty
Koc University
Rumelifeneri Yolu, Sariyer, 34450, Istanbul, Turkey

Phone: +90 212 338 1490

KOÇ UNIVERSITY

What is Convolutive Sparse BCA?

- **BCA: Extended ICA** for Bounded Signals ⇒ Allows Separation of Both Independent and Dependent Signals [5]
- **SBCA: Extension of BCA** [7] for Sparse Bounded Signals
- **Convolutive SBCA: Natural extension of SBCA** [2] for convolutive mixtures

Instantaneous BSS:

- Instantaneous Sparse BCA Setup previously introduced in [2]:
- $s_1, s_2, \ldots, s_p$: Source Signals
- $y_1, y_2, \ldots, y_q$: Mixture Signals
- $\mathbf{H}$: $q \times p$ Mixing System, where

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_q \end{bmatrix}}_{\mathbf{y}} = \mathbf{H} \underbrace{\begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_p \end{bmatrix}}_{\mathbf{s}}$$

- $\mathbf{W}$: $p \times q$ Separator System
- $z_1, z_2, \ldots, z_p$: Separator Outputs

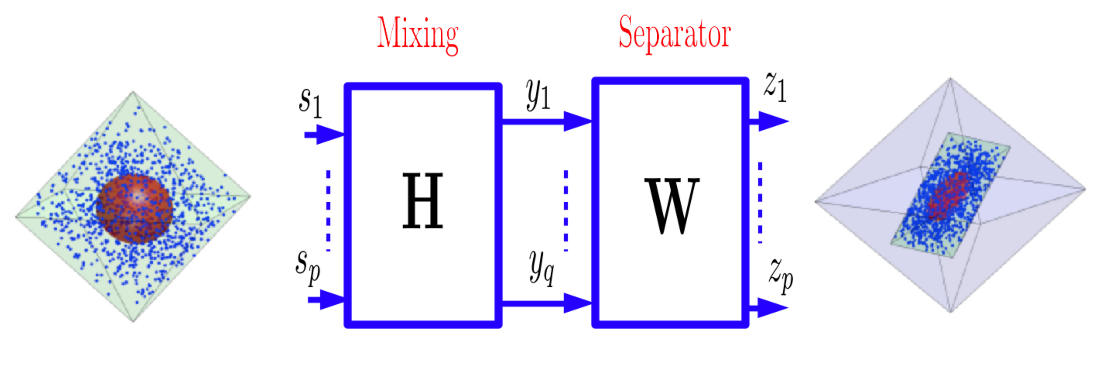

Figure 1: Geometric objects for the proposed Sparse BCA framework

Instantaneous Sparse BCA criterion given in [2]:

$$\text{maximize} \; \frac{\text{volume(Principal Hyper-ellipsoid)}}{\text{size(Bounding } l_1\text{-Norm-Ball)}}$$

The resulting instantaneous sparse BCA objective:

$$J(\mathbf{W}) = \frac{\sqrt{\det(\hat{\mathbf{R}}_z)}}{(\max_{n \in \{1,\ldots,L\}} \|\mathbf{z}(n)\|_1)^p}$$

Convolutive BSS:

- $s_1, s_2, \ldots, s_p$: Source Signals
- $y_1, y_2, \ldots, y_q$: Mixture Signals ($q \geq p$)
- $\tilde{\mathbf{H}}$: $q \times p$ Mixing System of order $K-1$, where

$$\tilde{\mathbf{H}} = [\mathbf{H}(0), \mathbf{H}(1), \ldots, \mathbf{H}(K-1)]$$

and

$$\underbrace{\begin{bmatrix} y_1(n) \\ y_2(n) \\ \vdots \\ y_q(n) \end{bmatrix}}_{\mathbf{y}(n)} = \tilde{\mathbf{H}} \underbrace{\begin{bmatrix} \mathbf{s}(n) \\ \mathbf{s}(n-1) \\ \vdots \\ \mathbf{s}(n-K+1) \end{bmatrix}}_{\tilde{\mathbf{s}}(n)}$$

- $\tilde{\mathbf{W}}$: $p \times q$ Separator System of order $M-1$, where

$$\tilde{\mathbf{W}} = [\mathbf{W}(0), \mathbf{W}(1), \ldots, \mathbf{W}(M-1)]$$

and

$$\underbrace{\begin{bmatrix} z_1(n) \\ z_2(n) \\ \vdots \\ z_p(n) \end{bmatrix}}_{\mathbf{z}(n)} = \tilde{\mathbf{W}} \underbrace{\begin{bmatrix} \mathbf{y}(n) \\ \mathbf{y}(n-1) \\ \vdots \\ \mathbf{y}(n-M+1) \end{bmatrix}}_{\tilde{\mathbf{y}}(n)}$$

- $z_1, z_2, \ldots, z_p$: Separator Outputs
- $\tilde{\mathbf{G}}$: Overall System of order $P-1$, where

$$\mathbf{G}(k) = \sum_{l=0}^{P-1} \mathbf{W}(l)\mathbf{H}(k-l), \; k = 0, \ldots, P-1.$$

- Basic Assumption: BCA's domain separability assumption [5]
- The goal is to obtain a separator matrix $\tilde{\mathbf{W}}$ such that the overall mapping

$$\tilde{\mathbf{G}} = \tilde{\mathbf{W}}\tilde{\mathbf{H}}$$

is equal to

$$\mathbb{G}(z) = \begin{bmatrix} \alpha_1 z^{-d_1} & 0 & \ldots & 0 \\ 0 & \alpha_2 z^{-d_2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \alpha_p z^{-d_p} \end{bmatrix} \mathbf{P} \quad (1)$$

- α_k, d_k : Non-zero real scalings and non-negative integer delays
- $\mathbf{P}$: Permutation matrix

The modified objective function of [2] for the separation of convolutive sparse mixtures:

$$J(\tilde{\mathbf{W}}) = \frac{\sqrt{\det(\hat{\mathbf{R}}_{\tilde{\mathbf{z}}_N})}}{(\max_{n \in \{N,\ldots,L_1\}} \|\tilde{\mathbf{z}}_N(n)\|_1)^{Np}}$$

Outline of the Proof:

- Rewrite the objective function, in terms of the argument $\mathbf{G}(k) = \sum_{l=0}^{P-1} \mathbf{W}(l)\mathbf{H}(k-l)$ for $k = 0, \ldots, P-1$, and the operator Γ_N such that $\Gamma_N(\tilde{\mathbf{G}})$ is a block Toeplitz matrix of dimension $(Np) \times (N+P-1)p$ whose first block row is $[\mathbf{G}(0), \mathbf{G}(1), \ldots, \mathbf{G}(P-1), \mathbf{0}, \ldots, \mathbf{0}]$ and first block column is $[\mathbf{G}(0), \mathbf{0}, \ldots, \mathbf{0}]^T$.

$$J(\tilde{\mathbf{G}}) = \frac{\sqrt{\det(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T)}}{(\max_{n \in \{N,\ldots,L_1\}} \|\Gamma_N(\tilde{\mathbf{G}})\tilde{\mathbf{s}}_{L_3}\|_1)^{Np}}. \quad (2)$$

where $L_3 = N + P - 1$.

- We can write the following inequalities:

$$J(\tilde{\mathbf{G}}) \leq \frac{\sqrt{\det(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T)}}{(\| [\|\Gamma_N(\tilde{\mathbf{G}})_{1,:}\|_1 \cdots \|\Gamma_N(\tilde{\mathbf{G}})_{Np,:}\|_1] \|_1/(Np))^{Np}} \quad (3)$$

$$\leq \frac{\sqrt{\det(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T)}}{\|\Gamma_N(\tilde{\mathbf{G}})_{1,:}\|_1 \|\Gamma_N(\tilde{\mathbf{G}})_{2,:}\|_1 \cdots \|\Gamma_N(\tilde{\mathbf{G}})_{Np,:}\|_1} \quad (4)$$

$$\leq \frac{\sqrt{\det(\Gamma_N(\tilde{\mathbf{G}})\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}\Gamma_N(\tilde{\mathbf{G}})^T)}}{\|\Gamma_N(\tilde{\mathbf{G}})_{1,:}\|_2 \|\Gamma_N(\tilde{\mathbf{G}})_{2,:}\|_2 \cdots \|\Gamma_N(\tilde{\mathbf{G}})_{Np,:}\|_2} \quad (5)$$

- Using Schur complement and Hadamard inequality for the nominator of (2), we obtain the inequality

$$\sqrt{\det(\hat{\mathbf{R}}_{\tilde{\mathbf{z}}_N})} \leq \prod_{m=1}^{Np} \| \Gamma_N(\tilde{\mathbf{G}})_{m,:} \|_2^2 \prod_{n=1}^{(P-1)p} \| \mathbf{Y}_{n,:} \|_2^2 \det(\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}}) \quad (6)$$

- The resulting inequality for the objective:

$$J(\tilde{\mathbf{G}}) \leq \prod_{n=1}^{(P-1)p} \| \mathbf{Y}_{n,:} \|_2 \det(\hat{\mathbf{R}}_{\tilde{\mathbf{s}}_{L_3}})^{1/2} \quad (7)$$

- **Result:** The upper bound for the objective $J(\tilde{\mathbf{G}})$ on the right hand-side of (7) is achieved if and only if $\mathbb{G}(z) = diag\left(\alpha_1 z^{-d_1}, \alpha_2 z^{-d_2}, \ldots, \alpha_p z^{-d_p}\right)\mathbf{P}$.

Iterative Algorithm for SBCA:

- We can write the iterative update equation using Clarke subdifferential [3] $J(\tilde{\mathbf{W}})$ as

$$\tilde{\mathbf{W}}^{(t+1)} = \tilde{\mathbf{W}}^{(t)} + \sigma^{(t)}\Big(\sum_{k=0}^{N-1} \mathbf{X}^{(t)}_{kp+1:(k+1)p,kq+1:(k+M)q} - Np\frac{\sum_{k=0}^{N-1} \mathbf{o}_{kp+1:(k+1)p,kq+1:(k+M)q}}{\max_{n \in \{N,\ldots,L_1\}} \|\tilde{\mathbf{z}}_N(n)\|_1}\Big)$$

where $\mathbf{o} = \text{sign}(\tilde{\mathbf{z}}_N(l^{(t)}))\tilde{\mathbf{y}}_{N+M-1}(l^{(t)})^T$ and $\mathbf{X} = \left(\Gamma_N(\tilde{\mathbf{W}})\hat{\mathbf{R}}_{\tilde{\mathbf{y}}_{N+M-1}}\Gamma_N(\tilde{\mathbf{W}})^T\right)^{-1}\Gamma_N(\tilde{\mathbf{W}})\hat{\mathbf{R}}_{\tilde{\mathbf{y}}_{N+M-1}}$.

Experimental Results:

- **Mixing**: The convolutive mixing system is i.i.d. Gaussian with order 3, and the separator is of order 4.
- **Comparison**: Castella's [4], Koldovský [8], Douglas' [6] algorithms.

First experiment: Source signals are synthetic sparse signal set given in the web-site of RIKEN Brain Science Institute [1].

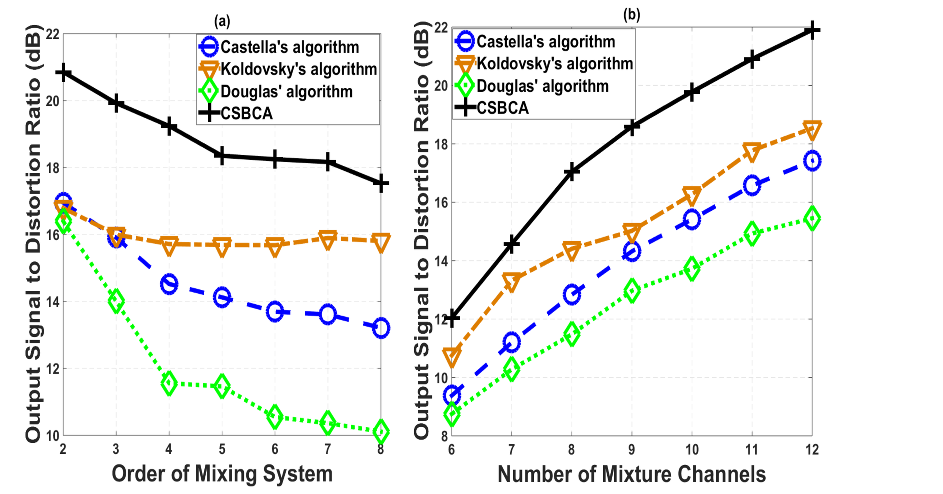

Figure 2: a)- Output SDR vs. mixing order for 5 sources and 10 mixture channels under SNR=20 dB. b)- Output SDR vs. number of mixture channels for 5 sources under SNR=20 dB.

Second experiment: Source signals are synthetic, sparse and dependent signals generated by using Copula-T distribution.

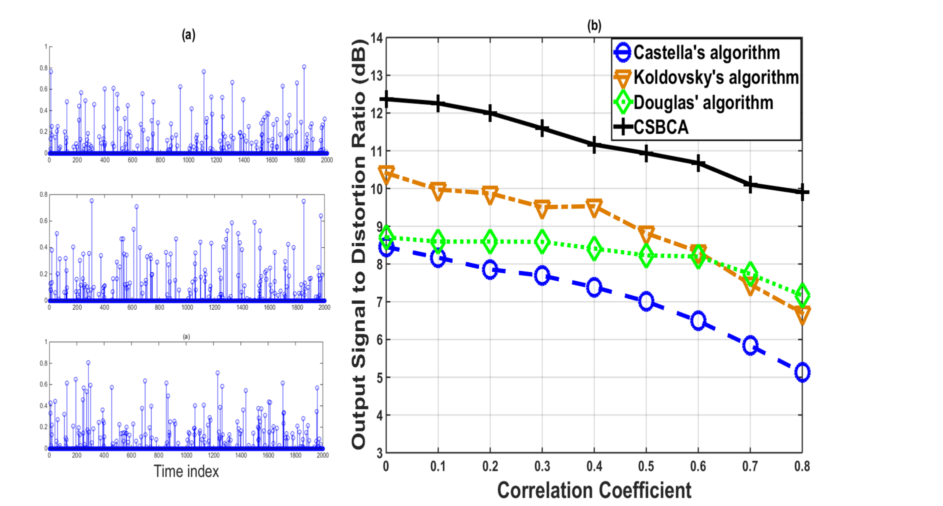

Figure 3: a)- Copula-T distributed random sparse sequences. b)- Output SDR vs. correlation for 3 sources under SNR=20 dB.

References

[1] S.-I. Amari A. Cichocki and K. Siwek. Icalab toolboxes. *http://www.bsp.brain.riken.jp/ ICALAB/ICALABSignalProc/benchmarks/*, 2003.

[2] Eren Babatas and Alper T. Erdogan. Sparse bounded component analysis. In *Machine Learning for Signal Processing (MLSP), 2016 IEEE 26th International Workshop on*, pages 1–6, 2016.

[3] Adil Bagirov, Napsu Karmitsa, and Marko M Mäkelä. *Introduction to Nonsmooth Optimization: theory, practice and software*. Springer, 2014.

[4] Marc Castella and Eric Moreau. A new method for kurtosis maximization and source separation. In *Acoustics Speech and Signal Processing (ICASSP), 2010 IEEE International Conference on (Code available at http://bass-db.gforge.inria.fr/bss_locate/)*, pages 2670–2673, 2010.

[5] Sergio Cruces. Bounded component analysis of linear mixtures: A criterion of minimum convex perimeter. *IEEE Transactions on Signal Processing*, 58(4):2141–2154, 2010.

[6] Scott C. Douglas, Hiroshi Sawada, and Shoji Makino. A spatio-temporal fastica algorithm for separating convolutive mixtures. In *ICASSP 2005*, pages 165–168, 2005.

[7] Alper T. Erdogan. A class of bounded component analysis algorithms for the separation of both independent and dependent sources. *IEEE Transactions on Signal Processing*, 61(22):5730–5743, 2013.

[8] Z. Koldovský and P. Tichavský. Time-domain blind audio source separation using advanced ica methods. In *Proceedings of 8th Annual Conference of the International Speech Communication Association (Interspeech 2007)*, pages 846–849, August 2007.