On design of optimal smart meter privacy control strategy against adversarial MAP detection

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Overview

1. Smart meter privacy problem

2. Design approach
   - Privacy model: Adversarial maximum a posteriori (MAP) detection
   - Stochastic optimal detection control strategy

3. Numerical study

4. Conclusion
Patterns in $\{\hat{H}_k\}$ can be used to infer, for example, religious, economic and social identities of users.

In Europe, GDPR regulates collecting, storing, or processing of data with sensitive personal information.
Privacy-by-design

Existing studies design EMU based on:
- **Information theory**: Variance\(^1\), Mutual information\(^2,3,4\) etc,
- **Detection theory**: Bayesian hypothesis testing adversary\(^5,6,7\).

Our previous work\(^6,7\) focused on including real ESS aspects in EMU design.

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Problem formulation

- **System model:** HMM characterized by \( (\mathcal{H}, \mathcal{X}, P_{H_k|H_{k-1}}, P_{X_k|H_k}) \).

- **Privacy model:** Adversarial maximum a posteriori (MAP) detection

\[
\hat{h}_1^N (y_1^N) = \arg\max_{h_1^N \in \mathcal{H}^N} P_{H_1^N, Y_1^N} (h_1^N, y_1^N)
\]

\[
= \arg\max_{h_1^N \in \mathcal{H}^N} \sum_{k=1}^{N} \log \left[ P_{H_k, Y_k|H_{k-1}} (h_k, y_k|h_{k-1}) \right].
\]

- **How to optimally control adversarial MAP detection performance?**

- **Design approach:** Stochastic optimal control of avg. detection cost, known as *Bayesian risk*, in EMU-unaware and -aware adversarial cases.
Optimal control of EMU-unaware MAP detection

- MAP estimate can be obtained using Viterbi (non-causal) algorithm.
- In the controller design, we compute a causal detection strategy $\zeta_k^*$ that achieves avg. Viterbi performance using dynamic programming:

  **Per-step reward:**
  \[
  r_k(x_k, \hat{h}_{k-1}^k) := \max \left[ \log \left[ P_{H_k, x_k | H_{k-1}} (\hat{h}_k, x_k | \hat{h}_{k-1}) \right], r_{\text{min}} \right],
  \]

  **Aggregate reward:**
  \[
  V_k(x_k, \hat{h}_{k-1}) := \max_{\hat{h}_k \in \mathcal{H}} \left[ r_k(x_k, \hat{h}_{k-1}^k) + \mathbb{E} \left[ V_{k+1}(X_{k+1}, \hat{h}_k) \right] \right].
  \]

- The optimal control strategy $\mu_k^*$ computed using the dynamic programming$^8$:

  **Per-step cost:**
  \[
  c_k(w_k, y_k, \zeta_k^*) := f_c(h_k, \zeta_k^*(y_k, \hat{h}_{k-1})),
  \]

  **Aggregate cost:**
  \[
  J_k(w_k) := \min_{y_k \in \mathcal{Y}} \left[ c_k(w_k, y_k, \zeta_k^*) + \mathbb{E} \left[ J_{k+1}(W_{k+1}) \right] \right].
  \]

- Discrete state and action spaces $\rightarrow$ **discrete optimization.**

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$^8$Controller state: $w_k = \{x_k, z_k, h_k, \hat{h}_{k-1} \}$
Optimal EMU-aware MAP detection

- The adversarial belief state $\hat{\pi}_k$ on the state $s_k = f_s(h_k, z_{k+1})$ is

$$
\hat{\pi}_k = \frac{M_k(y_k, \hat{h}_{k-1}, \mu_k) \hat{\pi}_{k-1}}{1^T \mathcal{H} M_k(y_k, \hat{h}_{k-1}, \mu_k) \hat{\pi}_{k-1}}; \quad [\hat{\pi}_k]_s = P_{S_k|Y_1}^k(s|y_1^k),
$$

where $M_k$ is belief transformation matrix function given by the HMM.

- The optimal detection strategy $\bar{\zeta}_k^*$ computed using dynamic programming$^9$

\begin{align*}
\text{Per-step reward:} & \quad \tilde{r}_k(\gamma_k, \hat{h}_k, \mu_k) := \max \left[ \log \left( \frac{a^T (y_k, \hat{h}_{k-1}, \mu_k) \hat{\pi}_{k-1}}{b^T (\hat{h}_{k-1}, \mu_k) \hat{\pi}_{k-1}} \right), r_{\min} \right], \\
\text{Aggregate reward:} & \quad \tilde{V}_k(\gamma_k, \mu_k) := \max_{\hat{h}_k \in \mathcal{H}} \left[ \tilde{r}_k(\gamma_k, \hat{h}_k, \mu_k) + \mathbb{E}[\tilde{V}_{k+1}(\Gamma_{k+1}, \mu_{k+1})] \right],
\end{align*}

where $a, b$ are vector functions given by the HMM.

$^9$ Control strategy $\mu_k : \mathcal{N} \rightarrow \mathcal{Y};$ Adversarial state: $\gamma_k := [y_k, \hat{h}_{k-1}, \hat{\pi}_{k-1}]$
Optimal control of EMU-aware MAP detection

Similarly, the optimal control strategy \( \bar{\mu}_k^* \) computed using the dynamic programming\(^{10}\):

- **Per-step cost:**
  \[
  \tilde{c}_k(\lambda_k, \mu_k, \bar{\zeta}_k^*) := f_c(h_k, \bar{\zeta}_k^*(\gamma_k, \mu_k)),
  \]

- **Aggregate cost:**
  \[
  \tilde{J}_k(\lambda_k) := \min_{\mu_k \in \mathcal{U}} \left[ \tilde{c}_k(\lambda_k, \mu_k, \bar{\zeta}_k^*) + \mathbb{E} [\tilde{J}_{k+1}(\Lambda_{k+1})] \right].
  \]

**Challenges:**

1. \( \gamma_k \) and \( \lambda_k \) contain \( \hat{\pi}_{k-1} \) \( \implies \) **continuous optimization.**
2. The aggregate adversarial reward \( \tilde{V}_k \) is piecewise concave w.r.t. \( \hat{\pi}_{k-1} \).

\(^{10}\) **Controller state:**
\[
\lambda_k = \{x_k, z_k, h_k, \hat{h}_{k-1}, \hat{\pi}_{k-1}\}
\]
Sub-optimal control: Adaptive-grid approximation algorithm

1. Find $Q$, the partitions of the simplex $\Delta_{|S|}$ using the hyperplanes
   \[ \{ \pi \in \Delta_{|S|} : (a_i - a_j)^T \pi = 0 \} \]
   for all possible vectors $a_i, a_j$ which gives per-step reward decision regions.

2. Recursively partition the simplex $\Delta_{|S|}$ using $Q$ and propagate them using all possible belief transformation matrices $M_k$.

3. Approximate each resulting partition with a finite number of points and solve the dynamic programming equation at these finite points.
Numerical study

- Simulation study: binary states; $|\mathcal{K}| = 6$; risk = detection prob.; 2000 MC simulations, $P_{X_k|H_k} = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}$, $P_{H_k|H_{k-1}} = \begin{bmatrix} 0.01 & 0.9 \\ 0.99 & 0.1 \end{bmatrix}$.

- EMU: Energy management unit
- $A_V$: Standard Viterbi algorithm
- $A_1$: EMU-unaware causal adversary
- $A_2$: EMU-aware causal adversary (regular grid approx.)
- $A_3$: EMU-aware causal adversary (proposed suboptimal approx.)
Conclusion

- We have presented the design of an optimal control against an adversarial MAP detection.
- The optimal control strategy against EMU-unaware adversary can be computed efficiently by solving discrete optimization problems.
- Whereas, the optimal control against EMU-aware adversary becomes non-convex due to piece-wise concave structure of Bellman’s equation. We presented a sub-optimal control strategy exploiting Bayesian evolution of belief state.
- Numerical study shows that the sub-optimal algorithm achieves close to the optimal performance.

Thank you!