Fast Adaptive Bilateral Filtering of Color Images

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IEEE International Conference on Image Processing, Taipei (2019)
Nonlinear edge-preserving smoothing\(^1\):

\[
g(i) = \eta(i)^{-1} \sum_{j \in \Omega} \omega(j) \phi(f(i - j) - f(i)) f(i - j),
\]

\[
\eta(i) = \sum_{j \in \Omega} \omega(j) \phi(f(i - j) - f(i)),
\]

where

- \( f \) and \( g \) are the input and output RGB images.
- \( f(i) \) and \( g(i) \) are vectors.
- \( \omega \) and \( \phi = \) Gaussian kernels with variance \( \rho^2 \) and \( \sigma^2 \).
- \( \Omega = \) Neighborhood for averaging.

\(^1\)Tomasi and Manduchi, 1998
Role of $\sigma$

Input.

Output, $\sigma = 30$.  

Output, $\sigma = 200$. 

Weights

Weights
Adaptation of $\sigma$

- $\sigma$ (width of range kernel) controls the extent of blurring.
- A fixed $\sigma$ either over or under smooths.
- Useful for controlling the blur in different regions, e.g., more blur to remove coarse textures in images.
- $\sigma$ is allowed to change at each pixel (a rule is required).
- Proposed for a couple of applications (for grayscale images):
  - Image sharpening\(^2\).
  - JPEG deblocking\(^3\).

\(^2\)Zhang and Allebach, 2008.
\(^3\)Zhang and Gunturk, 2009.
Adaptive bilateral filter (ABF)

- Make the width of the range kernel a function of \(i\).
- Moreover, allow center\(^4\) to be different from \(f(i)\).

\[
g(i) = \eta(i)^{-1} \sum_{j \in \Omega} \omega(j) \phi_i(f(i - j) - \theta(i)) f(i - j),
\]

\[
\eta(i) = \sum_{j \in \Omega} \omega(j) \phi_i(f(i - j) - \theta(i)) f(i - j).
\]

- However, a fixed spatial kernel is used.

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\(^4\)Zhang and Allebach, 2008.
Computation cost

- $O(\rho^2)$ computations per pixel.
- Higher $\rho$ (window size) is used for higher-resolution images.
- e.g. 60 seconds for a 2 megapixel image on a CPU.
- Real-time implementation is challenging.
- Fast approximation: Approximate the original formula and hope to speed it up, without appreciable loss of visual information.
Fast bilateral filtering

- Several fast algorithms for classical bilateral filtering (gray/color).
- Complexity does not scale with filter width ($O(1)$ implementation).
- Almost all fundamentally require the range kernel to be fixed.
- Filtering reduced to fast convolutions by approximating the range kernel.
- Rules out extension to ABF (range kernel is changing).
Our contribution

- Novel $O(1)$ algorithm for fast ABF of color images.
- Builds on a recently proposed algorithm for gray images.$^5$
- Trivial channel-by-channel extension to color images (3X cost).
- Filtering in RGB space?
- As explained later, this poses technical challenges.
- Core idea: Express filtering using local (weighted) histograms.$^6$

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$^5$Gavaskar and Chaudhury, 2019.
$^6$Mozerov and van de Weijer, 2015.
Local weighted histogram

- Local histogram at pixel $i$:
  \[ h_i(t) = \sum_{j \in \Omega} \delta(f(i - j) - t), \quad t \in \{0, \ldots, 255\}^3. \]

- $t = (t_r, t_g, t_b)$ and $\delta(t) = \delta(t_r) \delta(t_g) \delta(t_b)$.

- Local weighted histogram at pixel $i$:
  \[ h_i(t) = \sum_{j \in \Omega} \omega(j) \delta(f(i - j) - t), \quad t \in \{0, \ldots, 255\}^3. \]

- Interpretation: Spatially-weighted frequency of RGB value $t$. 
Reformulation of ABF

ABF in terms of local weighted histograms:

\[ g(i) = \eta(i)^{-1} \sum_t th_i(t) \phi_i(t - \theta(i)), \]

and

\[ \eta(i) = \sum_t h_i(t) \phi_i(t - \theta(i)), \]

where sum is over RGB values in the neighborhood of \( i \).
ABF for grayscale images can be similarly reformulated.

In grayscale, \( h_i(t) \) is a function of a scalar variable.

For fast algorithm, \( h_i(t) \) is approximated using polynomials\(^7\).

This gave closed-form Gaussian integrals.

Histogram approximation using fast convolutions (moment matching).

For color images, \( h_i(t) \) is a function of a vector variable.

Polynomial approximation is bad due to sparse data.

\(^7\)Gavaskar and Chaudhury, 2019.
Motivated by the approach in Mozerov and van de Weijer\textsuperscript{8}:

\begin{itemize}
\item $h_i(t)$ is constant over an interval $[a_i, b_i]$ (in $\mathbb{R}^3$).
\item $h_i(t)$ is zero elsewhere.
\item Summations are replaced by line integrals:
\begin{align*}
\hat{g}(i) &= \hat{\eta}(i)^{-1} \int_{[a_i, b_i]} t \phi_i(t - \theta(i)) \, dt,
\hat{\eta}(i) &= \int_{[a_i, b_i]} \phi_i(t - \theta(i)) \, dt.
\end{align*}
\end{itemize}

The integrals, and hence the filter, have a closed-form expression.

By clever choice of the interval, the computation becomes $O(1)$.

\textsuperscript{8}Mozerov and van de Weijer, 2015.
Novelty of our proposal

- In Mozerov and van de Weijer, the interval was chosen to be passing through $f(i)$.
  - having direction $\bar{f}(i) - f(i)$, where $\bar{f}(i) =$ mean value.

- This makes the algorithm $O(1)$, but is an ad-hoc choice.

- We choose the interval such that it captures linear trend of data.

- To do this, we use the covariance of the local weighted histogram.

- Our proposed algorithm is also $O(1)$. 
Choice of interval

- Covariance matrix:
  \[ C_i = \sum_{j \in \Omega} \omega(j) (f(i - j) - \bar{f}(i)) (f(i - j) - \bar{f}(i))^\top. \]

- Direction of \([a_i, b_i]\) = Largest eigenvector of the covariance matrix.

- This should give “best” linear approximation of the set of data points.

- Proposal:
  \[ [a_i, b_i] = \left[ \bar{f}(i) - c \sqrt{\lambda_i} q_i, \bar{f}(i) + c \sqrt{\lambda_i} q_i \right]; \]
  \((\lambda_i, q_i) = \text{Top eigenpair of } C_i,\)
  \(c = \text{Positive constant, decides length of the interval.}\)
Fast computation of interval endpoints

- Recall: \( a_i = \bar{f}(i) - c\sqrt{\lambda_i} \ q_i, \ b_i = \bar{f}(i) + c\sqrt{\lambda_i} \ q_i. \)

- We must find a fast method to compute the end points.

- \( O(1) \) Gaussian convolutions come to our rescue.

- \( \bar{f}(i) = \omega \ast f(i) \rightarrow 3 \) Gaussian convolutions.

- \((p, q)\)th entry of \( C_i = \omega \ast (f_p f_q)(i) - (\omega \ast f_p(i)) \ (\omega \ast f_q(i)). \)

- 6 additional Gaussian convolutions to compute \( C_i \)'s.
Fast computation of interval endpoints

- $(\lambda_i, \mathbf{q}_i)$ computed using power iterations method.

- Power iterations:
  - Initialize $\mathbf{q}_i$ as unit vector along $\bar{f}(i) - f(i)$.\(^9\)
  - Iterate: $\mathbf{q}_i \leftarrow C_i \mathbf{q}_i / \| \mathbf{q}_i \|$.\(^9\)

- In practice, just one iteration is enough.

- $\lambda_i = \mathbf{q}_i^\top C_i \mathbf{q}_i$.

- Overall, computation of $\mathbf{a}_i, \mathbf{b}_i$ requires $O(1)$ operations.

\(^9\)Direction used in Mozerov and van de Weijer, 2015.
Filter approximation

Recall:

\[ \hat{g}(i) = \hat{\eta}(i)^{-1} \int_{[a_i,b_i]} t \, \phi_i(t - \theta(i)) \, dt, \]
\[ \hat{\eta}(i) = \int_{[a_i,b_i]} \phi_i(t - \theta(i)) \, dt. \]

The integrals have closed-form expressions in terms of \( a_i, b_i \).

This was made possible due to the nature of the approximation.

As computation of \( a_i, b_i \) is \( O(1) \), computation of \( \hat{g}(i) \) becomes \( O(1) \).
Filter approximation

Closed-form expression (mean + first-order correction):

\[
\hat{g}(i) = \bar{f}(i) + (2 (\beta - \alpha e_1 e_2^{-1}) - 1) c \sqrt{\lambda_i q_i},
\]

where

\[
\alpha = \sigma(i) / c \sqrt{2\pi \lambda_i},
\]

\[
\beta = \frac{1}{2c \sqrt{\lambda_i}} q_i^\top (\theta(i) - \bar{f}(i) + c \sqrt{\lambda_i q_i}),
\]

\[
e_1 = \exp \left( -\frac{(1 - \beta)^2}{\pi \alpha^2} \right) - \exp \left( -\frac{\beta^2}{\pi \alpha^2} \right),
\]

\[
e_2 = \text{erf} \left( \frac{1 - \beta}{\sqrt{\pi \alpha}} \right) - \text{erf} \left( -\frac{\beta}{\sqrt{\pi \alpha}} \right).
\]

Main point: All computations are \(O(1)\).
Summary of the algorithm

1. Compute \( \omega \ast (f_p f_q), \omega \ast f_p \) for \( p, q = 1, 2, 3 \) using \( O(1) \) convolutions.

2. For each pixel \( i \),
   2.1 Populate \( C_i \) using the above convolved quantities.
   2.2 Estimate dominant eigenpair \((\lambda_i, q_i)\) by power iterations method.
   2.3 Compute \( \alpha, \beta, e_1, e_2 \) in the previous slide.
   2.4 Compute \( \hat{g}(i) \) using the formula in the previous slide.

Dominant cost = 9 Gaussian convolutions.
Application: Adaptive detail enhancement

Brief overview:

- **Objective:** Enhance details, but not to the same extent everywhere.

- More enhancement in regions which are more visually salient.

- Can be accomplished using the ABF$^{10}$.

- $\sigma(i)$ is decided using a saliency map.

- $\theta(i) = f(i)$.

- We use our proposed algorithm for color filtering.

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$^{10}$Ghosh et al., 2019.
Input (640 × 960).

Enhanced, $\rho = 5$.

Saliency map.

$\sigma$ map.

Application: JPEG deblocking

Brief overview:

▶ Objective: Smooth out blocking artifacts in JPEG-compressed images.

▶ For grayscale images, can be accomplished using ABF\textsuperscript{11,12}.

▶ We extend the same idea to color images.

▶ $\sigma(i)$ is decided using a technique proposed previously \textsuperscript{11}.

▶ $\theta(i) = f(i)$.

▶ We use our proposed algorithm for filtering.

\textsuperscript{11} Zhang and Gunturk, 2009.
\textsuperscript{12} Gavaskar and Chaudhury, 2019.
Input (512 × 512).

Deblocked, $\rho = 4$.

$\sigma$ map.

Original.

Timings: Brute-force = 8.4 sec., Proposed = 0.6 sec.
Application: Sharpening

Brief overview:

▶ Objective: Sharpen a blurred image containing fine noise grains.

▶ For grayscale images, can be accomplished using ABF\textsuperscript{13}.

▶ We extend the idea to color images.

▶ Both $\sigma(i)$ and $\theta(i)$ are decided using previously proposed techniques.

▶ We use our proposed algorithm for filtering.

\textsuperscript{13}Zhang and Allebach, 2008.
Input (1600 × 1200). Sharpened, $\rho = 4$.

Conclusion

- Proposed $O(1)$ algorithm for adaptive bilateral filtering of color images.

- First such algorithm to the best of our knowledge.

- Core idea: Approximate local histogram as uniform along direction of maximum variance.

- Achieves about $15 \times$ speedup with reasonable accuracy.

- Useful for detail enhancement, sharpening, and deblocking.

- Better accuracy and extension to non-Gaussian kernels?

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Research supported by EMR grant SERB/F/6047/2016-2017 from DST-SERB, Government of India.


Thanks for listening!