Iterative Beam Alignment Algorithms for TDD MIMO Systems
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Overview

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- Sparse mmWave model

Part 4: Conclusion
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- Acknowledgments
Part 1: Introduction
Introduction

- 5G technologies rely on **beamforming gains** to realize data rate requirements
  - **Millimeter-wave (mmWave):** Compensation for increased path and penetration loss in 25-100 GHz band
  - **Massive MIMO:** Multi-user beamforming in sub-6 GHz bands
- However: **Optimal beamforming weights depend on the channel matrix**

\[
y = z^* H f x + n
\]
Introduction

- With sufficiently small arrays,
  a) Use sounding sequences and feedback for each antenna
  b) Directly compute optimal beamformers (i.e. singular vectors of channel matrix)

- Problem: **Sounding approach is impractical with many antennas**

- Solution: **Beam-based sounding**
  - Users always transmit on beams
  - Acquire beamformers using a TDD beam alignment phase

- **Need for practical approaches to TDD-based beam alignment** (i.e. with additive noise, mmWave channel models)
  - Beamsweeping (codebook-based)
  - Greedy $\rightarrow$ ping-pong framework
Ping-pong beam alignment framework divides each channel use $k$ into two time slots.

Slot 1 (ping)

Node 1 sounds beam $f[k]$ as

$$y_o[k] = \sqrt{\rho_o} H f[k] + n_o[k]$$

Slot 2 (pong)

Node 2 sounds beam $z[k]$ as

$$y_e[k] = \sqrt{\rho_e} H^T z[k] + n_e[k]$$

Notation: $H$ — $M_r \times M_t$ channel matrix, $\rho_e, \rho_o$ — beam alignment SNR, $n_e[k], n_o[k]$ — complex additive white Gaussian noise.
Propose new beam alignment algorithms based on power method

- Works well for the noiseless case
- Convergence can slow down dramatically under additive noise
Prior work

- Precoding for sparse mmWave channels\(^1\)
- Hybrid beamforming\(^2\)
- Beamspace MIMO for mmWave systems\(^3\)
- Alternative eigenvalue iterations (i.e. Arnoldi iteration\(^4\))

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Part 2: Proposed Beam Alignment Algorithms
Sequential Least Squares (SLS) Power Method

Main Ideas
▶ Construct a least-squares (LS) estimate of the channel matrix using the sounding beams
▶ Compute greedy estimates of the singular vectors

Beamforming weights
\[
f[k] = \frac{\hat{H}_{e,k}^* z[k - 1]}{\|\hat{H}_{e,k}^* z[k - 1]\|_2}
\]
\[
z[k] = \frac{\hat{H}_{o,k} f[k]}{\|\hat{H}_{o,k} f[k]\|_2}
\]
Sequential Least Squares (SLS) Power Method

- Batch LS estimator:

\[ \hat{H}_{o,k} = \frac{Y_{o,k} (F_k)^\dagger}{\sqrt{\rho_o}} \]

- Requires full-rank observation matrix \( Y_{o,k} \)

- Instead, construct estimates sequentially:

\[ \hat{H}_{o,k} = f \left( \hat{H}_{o,k-1}, y_o[k], f[k] \right) \]
Computational complexity

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<tr>
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<th>Computational Count</th>
<th>Feedback</th>
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<tbody>
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$k_{\text{max}} = \text{Number of beam alignment ping-pong slots}$

$M = \max(M_t, M_r)$

- SLS Power Method performs very well at high costs (feedback and computational overhead)
Summed Power Method

- Transmit weights $f[k]
- Compute beamforming weights $z[k]

Main Ideas
- Derive beamforming weights as a function of the running sum of received observations
- Average over potentially noisy estimates during beam alignment

Beamforming weights

\[
\begin{align*}
z[k + 1] &= \beta_k [y_o[k] + y_o[k - 1] + \cdots + y_o[0]] \\
&= \beta_k s_o[k] \\
\end{align*}
\]

\[
\begin{align*}
f[k + 1] &= \alpha_k [\overline{y}_e[k] + \overline{y}_e[k - 1] + \cdots + \overline{y}_e[0]] \\
&= \alpha_k s_e[k] \\
\end{align*}
\]
Summed Power Method

- Normalize using $\ell_2$-norm

\[
\alpha_k = \frac{1}{\|s_e[k]\|_2}, \quad \beta_k = \frac{1}{\|s_o[k]\|_2}
\]

- Repeated conjugation and retransmission like in the simple power iteration
- Averaging observations reduces the effects of additive noise
- Little overhead
- No feedback necessary
### Computational complexity

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$k_{\text{max}} = \text{Number of beam alignment ping-pong slots}$

$M = \max(M_t, M_r)$

**How to combine the positive properties of both techniques?**

**What are the tradeoffs?**

15/22
Least-squares initialized Summed Power Method (LISP method)\(^5\)

- Idea: “prime” the beamformer estimates up to period \(k_{\text{switch}}\) with the SLS method, then switch to the Summed Power Method

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Part 3: Simulation Results
### Overview

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<td>BIMA(^6)</td>
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<tr>
<td>BSM(^7)</td>
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**Metrics:**

- Effective channel gain \( |z^*[k]Hf[k]|^2 \)
- Chordal distance to dom. sing. vector \( \phi_k = \cos^{-1} \left( |f_{\text{opt}}^*f[k]| \right) \)

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IID Rayleigh fading model

Parameters:

\[ \rho_e = \rho_o = -10 \text{ dB}, \quad M_r = 4, \quad M_t = 32, \quad k_{\text{switch}} = \max(M_r, M_t) \]
Sparse mmWave model

Parameters:
\[ \rho_e = \rho_o = -10 \text{ dB}, \quad M_r = 4, \quad M_t = 32, \]
\[ k_{\text{switch}} = \max(M_r, M_t) \]

Channel model:
\[ \lambda/2\text{-spaced ULAs}, \quad f_c = 28 \text{ GHz}, \]
\[ K = 3 \text{ dominant clusters, one path/cluster} \]
Future research

- Analytical framework for convergence analysis as function of SNR, antenna dimensions, etc.
- Impact of noisy feedback for SLS method
- Time-varying channels
- Application to hybrid beamforming systems
- Applications to machine learning, principal component analysis-type problems
Acknowledgments

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—Begin Backup Slides—
With the sequential algorithm, node 2 computes its estimate according to the update equation

\[
\hat{H}_{o,k} = \hat{H}_{o,k-1} + \left( \frac{y_o[k]}{\sqrt{\rho_o}} - \hat{H}_{o,k-1} f[k] \right) K_{o,k} \tag{1}
\]

where

\[
K_{o,k} = \frac{f^*[k] C_{o,k-1}}{1 + f^*[k] C_{o,k-1} f[k]} \tag{2}
\]

and

\[
C_{o,k} = C_{o,k-1} (I - f[k] K_{o,k}) \tag{3}
\]
Impact of Antenna Dimensions

I.I.D Rayleigh fading model

Sparse mmWave model

Parameters:

\( \rho_e = \rho_o = -10 \text{ dB}, \quad M_r = 4, \quad M_t \in \{6, 8, \ldots, 64\}, \quad k_{\text{switch}} = \max(M_r, M_t), \quad 100 \text{ ping-pong slots} \)
Impact of $k_{\text{switch}}$

$\rho_e = \rho_o = 0 \text{ dB and } k_{\text{max}} = 100$

$\rho_e = \rho_o = -10 \text{ dB and } k_{\text{max}} = 400$

Parameters:

$M_r = 4, M_t = 32$
IID Rayleigh fading model

Parameters:

\[ \rho_e = \rho_o = 0 \text{ dB}, \ M_r = 4, \ M_t = 32, \ k_{\text{switch}} = \max(M_r, M_t) \]
I.I.D. Rayleigh fading model

![Graphs showing the convergence of the sum of powers and the average magnitude squared of the phase over iterations.]

Parameters:
\[ \rho_e = \rho_o = 20 \text{ dB}, \quad M_r = 4, \quad M_t = 32, \quad k_{\text{switch}} = \max(M_r, M_t) \]
Beam Pattern evolution

Beam pattern of $f[k]$ vs. beam pattern of $f_{opt}$

Parameters:

$\rho_e = \rho_o = -10$ dB, $M_r = 4$, $M_t = 32$, 200 ping-pong slots