Dynamic Power Allocation for Smart Grids via ADMM

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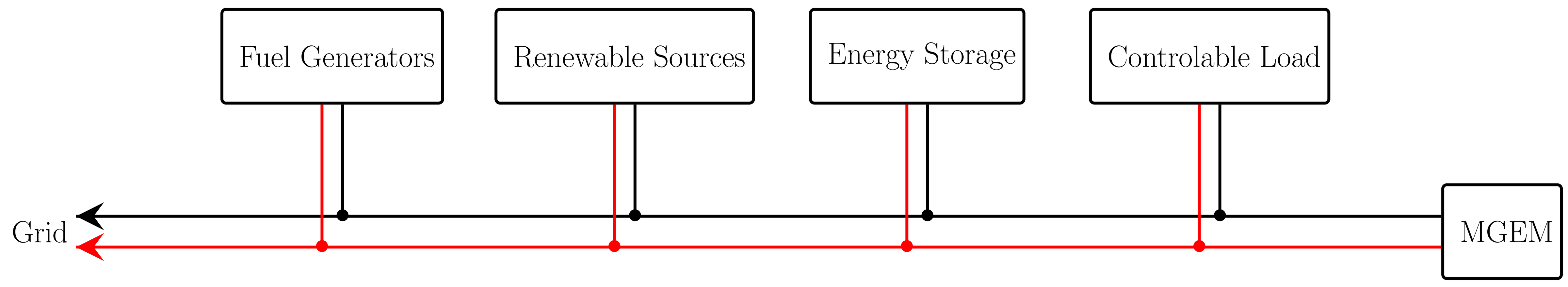


Decentralized infrastructure of a micro-grid with communications (black) and energy flow (red) networks. MGEM: Micro-grid energy manager

Dynamic Economic Dispatch

$$\min_{\{\mathbf{p}_i \in \mathbb{R}^p\}_{i=1}^N} \sum_{i=1}^{M} C_i(\mathbf{p}_i, k) - \sum_{i=M+1}^{N} U_i(\mathbf{p}_i, k)$$
$$\text{s.t. } \underline{\mathbf{p}}_i^{[k]} \leq \mathbf{p}_i \leq \overline{\mathbf{p}}_i^{[k]},\ i = 1, \ldots, N$$
$$\sum_{i=1}^{N} \mathbf{p}_i = \mathbf{P}^{[k]}$$

Assumption 1 (Regularity Conditions)**.** *Each of the* ***cost functions*** $C_i(\cdot, k)$ *is* $\mu-$*strongly convex and has* $L-$*Lipschitz continuous gradient. Analogously, each* ***utility function*** $U_i(\cdot, k)$ *is* $\mu-$*strongly concave and has* $L-$*Lipschitz continuous gradient.*

Assumption 2 (Bounded deviations)**.** *The primal dual optimal point from one iterate to the next deviates a bounded amount, i.e.* $\|\mathbf{p}^\star(k) - \mathbf{p}^\star(k+1)\| \leq \Delta p$ *and* $\|\boldsymbol{\lambda}^{\star[k]} - \boldsymbol{\lambda}^{\star[k+1]}\| \leq \Delta\lambda\ \forall k$.

What is desirable in a time varying set-up?

- If changes are fast, only one iteration per problem change
- Feasibility guarantees at each iteration
- Remain close to the optimal point

What do we require?

- For $\lim_{k\to\infty} \|\mathbf{p}^{[k]} - \mathbf{p}^{\star[k]}\| \leq C(\Delta p, \Delta\lambda)$ under the bounded deviations assumption we require Q-linear convergence of the method
- Strong coordination to guarantee per iteration constraint fulfilment

ADMM friendly reformulation

$$\min_{\{\mathbf{p}_i\}_{i=1}^N, \{\mathbf{q}_i\}_{i=1}^N} \sum_{i=1}^{N} f_i(\mathbf{p}_i) \triangleq$$
$$\sum_{i=1}^{M} C_i(\mathbf{p}_i, k) - \sum_{i=M+1}^{N} U_i(\mathbf{p}_i, k)$$
$$\text{s.t.} \underline{\mathbf{p}}_i^{[k]} \leq \mathbf{q}_i \leq \overline{\mathbf{p}}_i^{[k]},\ i = 1, \ldots, N$$
$$\sum_{i=1}^{N} \mathbf{q}_i = \mathbf{P}^{[k]},$$
$$\mathbf{p}_i = \mathbf{q}_i,\ i = 1, \ldots, N.$$

Advantages of the problem formulation:

- For fixed $\{\mathbf{q}_i\}_{i=1}^N$ the problem is smooth with strongly convex objectives in $\{\mathbf{p}_i\}_{i=1}^N$
- For fixed $\{\mathbf{p}_i\}_{i=1}^N$ the problem projects over the feasible set defined by the constraints. We will denote it by $\mathcal{Q}^{[k]}$.

ADMM with Total Feasibility

Initialize $\{\mathbf{p}_i\}_{i=1}^N$ and $\{\boldsymbol{\lambda}\}_{i=1}^N$. Set $k = 0$

Each node i obtains $f_i(\cdot, k+1)$, $\underline{\mathbf{p}}_i^{[k+1]}$, $\overline{\mathbf{p}}_i^{[k+1]}$, MGEM obtains $\mathbf{P}^{[k+1]}$

MGEM and nodes jointly compute

$$\{\mathbf{q}_i^{[k+1]}\}_{i=1}^N = \arg \min_{\{\mathbf{q}_i\}_{i=1}^N} g(\mathbf{q}, k)$$
$$\text{s.t.} \underline{\mathbf{p}}_i^{[k+1]} \leq \mathbf{q_i} \leq \overline{\mathbf{p}}_i^{[k]}$$
$$\sum_{i=1}^{N} \mathbf{q}_i = \mathbf{P}_i^{[k+1]},$$

$g(\mathbf{q}, k) \triangleq \frac{1}{2}\sum_{i=1}^N \left\|\mathbf{q}_i - \left(\mathbf{p}_i^{[k]} + \frac{\boldsymbol{\lambda}^{[k]}}{\rho}\right)\right\|^2$. See "Cooperative Projection"

Each node computes

$$\mathbf{p}_i^{[k+1]} = \arg \min_{\mathbf{p}_i} f_i(\mathbf{p}_i, k+1) + \boldsymbol{\lambda}_i^{[k]T} \mathbf{p}_i + \frac{\rho}{2}\left\|\mathbf{p}_i - \mathbf{q}_i^{[k+1]}\right\|^2$$

$$\boldsymbol{\lambda}_i^{[k+1]} = \boldsymbol{\lambda}_i^{[k]} + \rho\left(\mathbf{p}_i^{[k+1]} - \mathbf{q}_i^{[k+1]}\right)$$

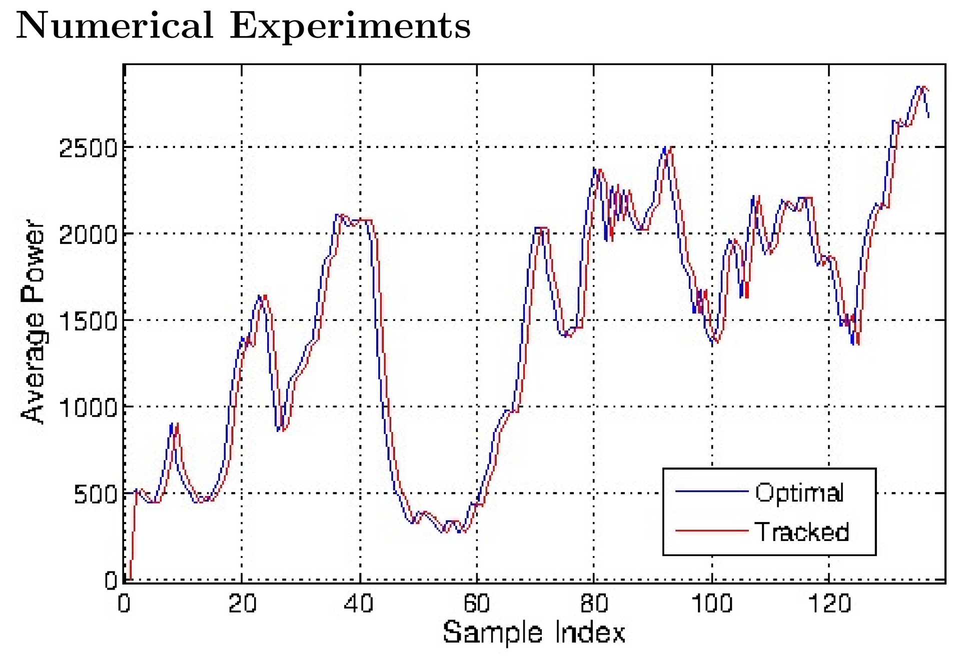
Tracking statement

The proposed algorithm generates a sequence of iterates $\{\mathbf{q}^{[k]}, \mathbf{p}^{[k]}\}$ that fulfills

$$\limsup_{k\to\infty} \|\mathbf{p}^{[k]} - \mathbf{p}^{\star[k]}\| \leq c_1$$
$$\limsup_{k\to\infty} \|\mathbf{q}^[k] - \mathbf{q}^{\star[k]}\| \leq c_2$$

where $c_1 \triangleq \frac{g}{\sqrt{1+\delta_{\max}}-1}$, $c_2 \triangleq 3c_1^2 + \frac{1}{\rho}g^2 + \frac{3}{\sqrt{\rho}}c_1 g$, $\delta_{\max} \triangleq \frac{1}{\sqrt{L/\mu}}$ and $g \triangleq \sqrt{\rho(\Delta p)^2 + \frac{1}{\rho}(\Delta\lambda)^2}$, and the sequence $\{\mathbf{q}^{[k]}\}$ is always primal feasible.

Numerical Experiments



Cooperative Projection

Nodes compute

$$\mathbf{m}_i = \left(\mathcal{P}_{\mathcal{Q}_i^{[k+1]}} - \mathcal{I}\right)(\mathbf{p}_i^{[k]} + \frac{\boldsymbol{\lambda}_i^{[k]}}{\rho})$$

Transmit $\mathcal{P}_{\mathcal{Q}_i^{[k+1]}}(\mathbf{p}_i^{[k]} + \frac{\boldsymbol{\lambda}_i^{[k]}}{\rho})$ MGEM receives

$$\mathbf{s}^{[k]} = \sum_{i=1}^{N} \mathcal{P}_{\mathcal{Q}_i^{[k+1]}}(\mathbf{p}_i^{[k]} + \frac{\boldsymbol{\lambda}_i^{[k]}}{\rho})$$

MGEM transmits $\mathbf{b}^{[k]} \triangleq \text{sign}(-\mathbf{s}^{[k]} + \mathbf{P}^{[k+1]})$

for $j = 1, \ldots, p$ **do**

  if $\text{sign}(b^{[k]}(j)) = \text{sign}(m_i(j)) | m_i(j) = 0$ **then**

    if $m_i(j) > 0$ **then**

      Node transmits $(m_i(j), \overline{x}_{ij})$, $\overline{x}_{ij} = \overline{p}_i(j) - \mathcal{P}_{\mathcal{Q}_i^{[k+1]}}(\mathbf{p}_i^{[k]} + \frac{\boldsymbol{\lambda}_i^{[k]}}{\rho})(j)$

    end if

    if $m_i(j) < 0$ **then**

      Node transmits $(m_i(j), \underline{x}_{ij})$, $\underline{x}_{ij} = \underline{p}_i(j) - \mathcal{P}_{\mathcal{Q}_i^{[k+1]}}(\mathbf{p}_i^{[k]} + \frac{\boldsymbol{\lambda}_i^{[k]}}{\rho})(j)$

    end if

    if $m_i(j) = 0$ **then** node tx $(0, \overline{x}_{ij})$ if $b_i(j) > 0$ and $\underline{x}_{ij}$ otherwise.

    end if

  end if

end for

$\mathcal{T}(j)$ set of nodes transmits regarding component j

MGEM solves

$$\min_{\{\Delta q_{ij} i \in \mathcal{T}(j),\, j=1,\ldots,p\}} \sum_{ij} \|\Delta q_{ij} + m_i(j)\|^2$$
$$\text{s.t.} \underbrace{0 \leq \Delta q_{ij} \leq \overline{x}_{ij}}_{\text{if } b_i^{[k]}(j)>0}$$
$$\underbrace{\underline{x}_{ij} \leq \Delta q_{ij} \leq 0}_{\text{if } b_i^{[k]}(j)<0}$$

MGEM transmits Δq_{ij}

Each node computes $q_i^{[k+1]}(j) = \Delta q_{ij} + \mathcal{P}_{\mathcal{Q}^{[k+1]}}(\mathbf{p}^{[k]} + \frac{\boldsymbol{\lambda}^{[k]}}{\rho})$

Information Exchange

Each iteration requires

- In worse case scenario, exchange of $3N$ $p-$sized real vectors and broadcast 1 p sized binary vector as compared to $3N$ $p-$sized real vectors if all information is sent to MGEM.
- this scenario will occur when no user lays in the boundary of the constraint set in any component.