SUREMap: Predicting Uncertainty in CNN-Based Image Reconstructions Using Stein’s Unbiased Risk Estimate

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Motivation (1)

• Convolutional neural networks (CNN) have emerged as a powerful tool for solving compressive sensing (CS) reconstruction problems.
• However, CNNs are black boxes.
Motivation (2)

• Expected mean squared error (MSE) is the gold standard for evaluating a CS reconstruction algorithm.

• Computing MSE requires the ground truth, which defeats the point of reconstruction in the first place.

\[ MSE = \frac{1}{n} ||\hat{x} - x||^2 \]

Squared difference \((\hat{x} - x)^2\)
Motivation (3)

• We can estimate MSE without requiring the ground truth using Stein’s Unbiased Risk Estimate (SURE) for CS reconstruction with Approximate Message Passing (AMP) framework.

• SURE works because AMP decouples the CS reconstruction into a series of Gaussian denoising problems.

Squared difference $(\hat{x} - x)^2$

SUREMap

Estimate without $x$
SURE with Denoising-based AMP (D-AMP) (1)

• Problem setting: $y = Ax + \eta$, estimate $x$ with $\hat{x}$.
  • $x \in \mathbb{R}^n$ ground truth, $y \in \mathbb{R}^m$ measurement, $m < n$.
  • $\eta \in \mathcal{N}(0, \sigma^2 I_m)$ Gaussian noise.

• At iteration $t$ of D-AMP, we solve $r_t = x + \eta_t$. The solution is $f(r_t) = \hat{x}$ where $f$ is a (possibly CNN-based) denoiser.
  • $\eta_t \in \mathcal{N}(0, \sigma_t^2 I_m)$ Gaussian noise.
SURE with Denoising-based AMP (D-AMP) (2)

• \(MSE = \frac{1}{n} \| f(r_t) - x \|^2\)

• \(SURE = \frac{1}{n} \| f(r_t) - r_t \|^2 + \frac{2\sigma_t^2}{n} \text{div}_{r_t}(f(r_t)) - \sigma_t^2\)

• Calculate the divergence using a Monte-Carlo estimate.

\[
\text{div}_{r_T}(\hat{x}) \approx \frac{1}{K} \sum_{k=1}^{K} \frac{1}{\epsilon} b_k^T (f(r_T + \epsilon b_k) - f(r_T)) \quad b_k \sim \mathcal{N}(0, I_n)
\]

• \(\mathbb{E}[MSE] = \mathbb{E}[SURE]\)
Patch-wise Calculation

- Average overlapping patches of SURE to obtain a SURE map.
- SURE map is equivalent to lowpass-filtered map of squared error.
SURE with Denoising-based VDAMP (D-VDAMP) (1)

• VDAMP (Millard et al.) extends AMP to variable density Fourier measurements as in MRI.

• Problem setting: \( y = M(Fx + \eta) \), estimate \( x \) with \( \hat{x} \).
  • \( M \) undersampling mask, \( F \) Fourier transform

• At each iteration of D-VDAMP, we solve

\[
\mathbf{r}_t = x + \eta_t \quad \eta_t \sim \mathcal{CN}(0, \mathbf{\Psi}^t \text{diag}(\tau_t) \mathbf{\Psi})
\]

to obtain \( f(r_t) = \hat{x} \).
SURE with Denoising-based VDAMP (D-VDAMP) (2)

- \(MSE = \frac{1}{n} ||f(r_t) - x||^2\)

- SURE
  \[S(\hat{x}, r_T) = ||\hat{x} - r_T||^2 - \sum_{i=1}^{n} \tau_T^{(i)} u = \Psi \text{diag} \left( \frac{1}{2} \tau_t \right)^{-1} \Psi^T r_T\]
  \[+ \frac{2}{n} \left( \text{div}_R(u) (\Re(\hat{x})) + \text{div}_S(u) (\Im(\hat{x})) \right)\]

- \(\mathbb{E}[MSE] = \mathbb{E}[SURE]\)
Experimental Results

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<th>SURE</th>
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Accuracy-Resolution Tradeoff

![Graph showing the tradeoff between accuracy and resolution.](image)

- **Ground truth**
- **Reconstruction**

The graph illustrates the relative error as a function of patch width, with different lines representing different values of $k$. The `MSE` and `SURE` values are shown for patch widths of 1 and 16.
Conclusion

• We can estimate per-pixel MSE of CS reconstruction with AMP + black-box denoiser without requiring ground truth.
• The accuracy-resolution tradeoff is a limitation to our approach.
• Usage of SURE heatmaps:
  • Inform end-users about the reliability of image reconstructions.
  • Serve as supplementary information for an artifact-removal algorithm.
  • Guide an adaptive sampling strategy.