Chance Constrained Optimization of Distributed Energy Resources via Affine Policies

Krishna Sandeep Ayyagari, Nikolaos Gatsis, and Ahmad Taha

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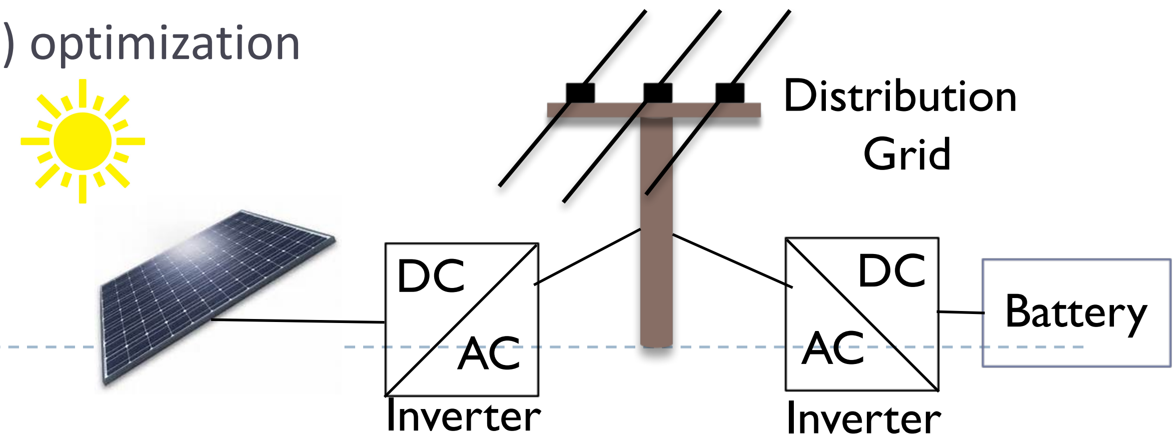
Opportunities in distribution systems

- Distribution systems envisioned to accommodate renewable distributed generation (DG)
- Challenge: Uncertainty and intermittency in renewable DG
- Stochasticity in renewable DG renders voltage profile uncertain
  - Potentially causing over- and under-voltage conditions
- Resources to mitigate uncertainty
  - Reactive power generated or consumed by photovoltaic (PV) inverters
  - Distributed storage: charge/discharge and reactive power support
- Limit the probability of nodal voltages violating specification
  - Chance constrained (CC) optimization



Prior art: Stochastic optimization in DN

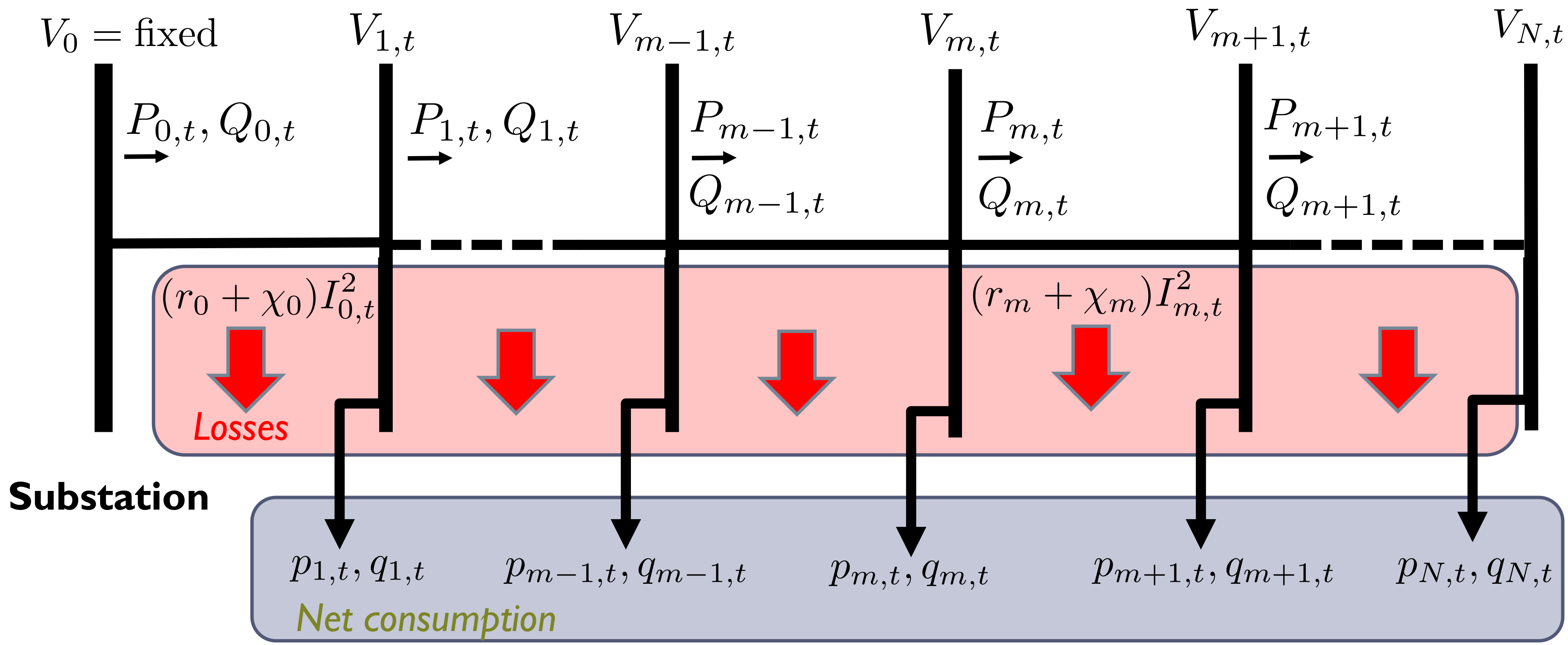
- Stochastic optimization in distribution systems (no chance constraints)

  [Kekatos et al. ’15] [Dall’Anese et al. ’15] [Wang et al. ’16] [Bazrafshan-Gatsis ’17]
- Chance constraints are typically nonconvex; three major approaches
  - Special distributions for the uncertainty (e.g., Gaussian) can lend tractability when the underlying model is linear
    - Earlier works on transmission networks [Sjodin et al. ’12][Bienstock et al. ’14]
    - Nonconvex model due to power flows in distribution networks [Cao et ’13]
  - Conservative convex approximations, e.g., using the conditional value-at-risk (CVaR) [Summers et al. ’15]
    - Distributions networks [Bazrafshan-Gatsis ’14] [Dall’Anese, Baker, Summers ’16]
    - No assumption on the distribution
  - Scenario approach [Calafiore-Campi ’06]
    - Can be conservative [Zhang et al. ’13]
    - Conditioning on recent observations can alleviate drawbacks [Bolognani et al. ’17]

Prior art: Control policies

- In regards to the type of control policy, there are three approaches
  - One-size-fits-all decision: Compute a single resource allocation that will work for all realizations of the uncertainty (typical in earlier works)
  - Scenario-dependent decisions: Consider discrete scenarios of the uncertainty, find one control action for each scenario
    - Typical with CVaR approaches
    - Increases the number of optimization variables
  - Affine policies: Control action is linear in the uncertainty
    - Transmission networks [Bienstock et al. '14] [Summers et al. '15], robust control of distr. systems [Lin-Bitar], building climate control [Oldewurtel et al. '08-'10]
- This work: Voltage regulation via chance constraints
  - Assumes Gaussianity, optimizes an affine policy
  - Reactive power from PV inverters; real and reactive power from storage
  - Minimize thermal losses

Simplified DistFlow equations



Approximations

1. Losses negligible
2. Voltage drop very small

[Baran-Wu '89]

$$P_{m-1,t} = P_{m,t} + p_{m,t} \quad (m = 0, \ldots, N-1;\ t = 1, \ldots, T)$$

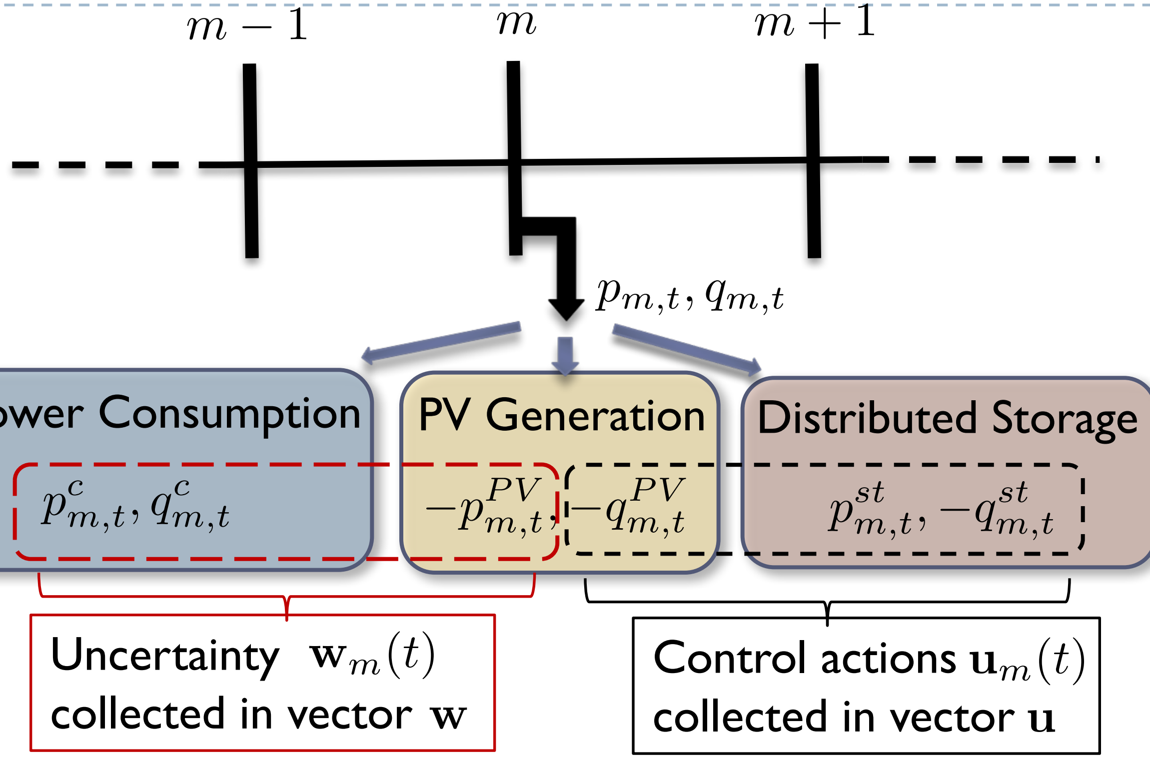
$$Q_{m-1,t} = Q_{m,t} + q_{m,t}$$

$$V_{m+1,t} = V_{m,t} - 2(r_m P_{m,t} + \chi_m Q_{m,t})$$

$$P_{N,t} = Q_{N,t} = 0 \quad (t = 1, \ldots, T)$$

$V_{m,t}$ = squared voltage magnitude

Uncertain vs. decision variables



- Vector $\mathbf{v}$ collects nodal voltages for all time slots
- Simplified DistFlow equations imply $\mathbf{v} = \mathbf{D}\mathbf{u} + \mathbf{E}\mathbf{w} + V_0\mathbf{1}$
- Vector $\mathbf{w}$ assumed Gaussian, $\mathbf{w} \sim \mathcal{N}(\bar{\mathbf{w}}, \mathbf{\Sigma})$; Cholesky fact. $\mathbf{\Sigma} = \mathbf{L}\mathbf{L}^\top$
- Reasonable assumption when $\mathbf{w}$ modeled as forecasted value + error

PV injection model

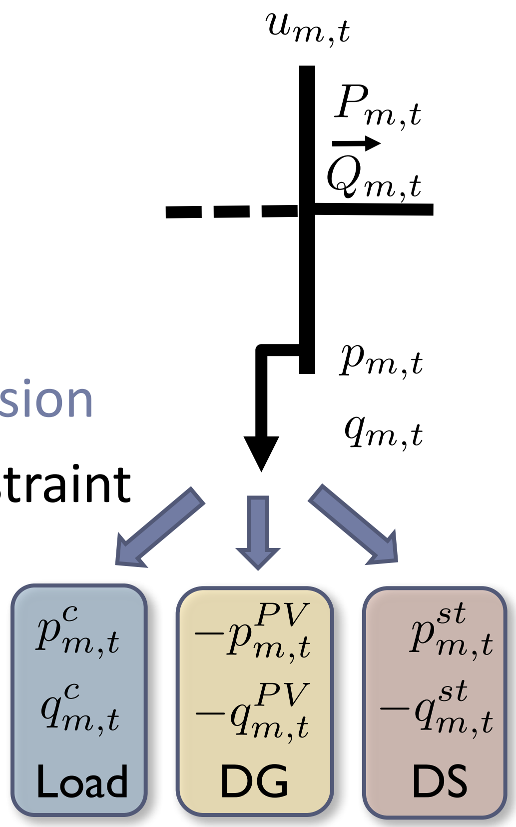
- Maximum real and apparent power capacities $p_{m,\max}^{PV}, S_{m,\max}^{PV}$

$$p_{m,t}^{PV} \leq p_{m,\max}^{PV} \quad (m = 1, \ldots, N;\ t = 1, \ldots, T)$$

- Inverter sizing to effect reactive power control

$$S_{m,\max}^{PV} > p_{m,\max}^{PV}$$

[Turitsyn, Šulc, Backhaus, Chertkov '10-'11]



- Reactive power $q_{m,t}^{PV}$ *generated or consumed*: decision
- Coupled with the uncertainty through inverter constraint

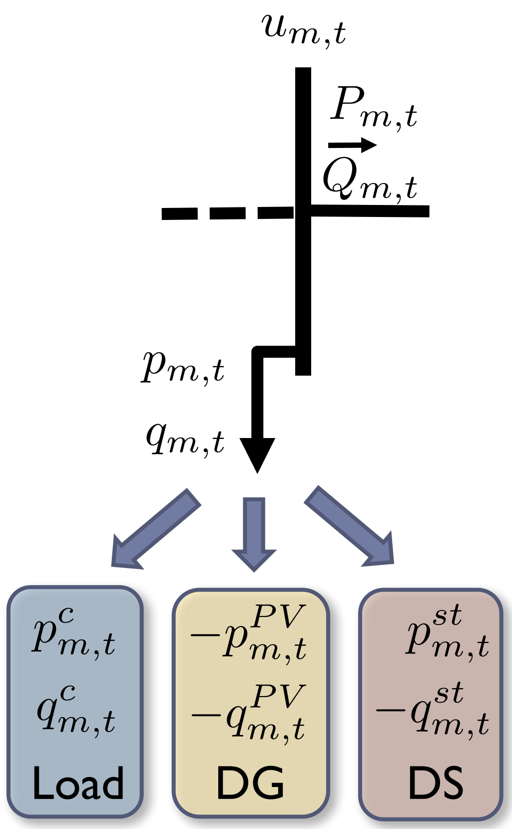
$$(q_{m,t}^{PV})^2 + (p_{m,t}^{PV})^2 \leq (S_{m,\max}^{PV})^2$$

Storage model

- Charge or discharge with limits $-p^{st}_{m,\max} \leq p^{st}_{m,t} \leq p^{st}_{m,\max}$
- Time slot duration δ; energy stored in the beginning of slot $x_{m,t}$

$$x_{m,t+1} = x_{m,t} + \delta p^{st}_{m,t}$$

- Initial condition $x_{m,1}$ known
- Storage capacity limit $0 \leq x_{m,t} \leq x_{m,\max}$
- Terminal constraint $x_{m,T+1} \geq \underline{x}_m$
- Storage inverter sizing $S^{st}_{m,\max} > p^{st}_{m,\max}$



- Reactive power provided by storage unit $q^{st}_{m,t}$

$$(p^{st}_{m,t})^2 + (q^{st}_{m,t})^2 \leq (S^{st}_{m,\max})^2$$

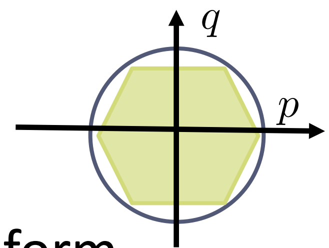
Quadratic objective, linear constraints

- Objective: Minimize thermal losses

$$\sum_{t=1}^{T}\sum_{m=0}^{N-1} r_i \frac{P_m^2(t)+Q_m^2(t)}{V_0} = \mathbf{u}^\top \mathbf{Q}\mathbf{u} + \mathbf{w}^\top \mathbf{R}\mathbf{w} + \mathbf{u}^\top \mathbf{S}\mathbf{w} + \mathbf{w}^\top \mathbf{S}^\top \mathbf{u}$$

- Voltages $\mathbf{v} = \mathbf{D}\mathbf{u} + \mathbf{E}\mathbf{w} + V_0\mathbf{1}$
- Storage states $\mathbf{x} = \mathbf{A}\mathbf{x}(1) + \mathbf{B}\mathbf{u}$
- Storage input, state, and terminal bounds $\mathbf{F}\mathbf{x} \leq \boldsymbol{\phi}, \mathbf{G}\mathbf{u} \leq \boldsymbol{\gamma}$
- Inner linear approx. of inverter constraints using polygon with ℓ facets

$$\begin{aligned} \boldsymbol{\Xi}_1\mathbf{w} + \boldsymbol{\Xi}_2\mathbf{u} &\leq \boldsymbol{\psi} \\ \mathbf{Z}\mathbf{u} &\leq \boldsymbol{\zeta} \end{aligned}$$



- Voltage regulation constraints $\mathbf{v}_{\min} \leq \mathbf{v} \leq \mathbf{v}_{\max}$ in matrix form

$$\mathbf{K}\mathbf{v} \leq \boldsymbol{\kappa}$$

Chance constrained optimization

- Uncertainty renders nodal voltages random

$$\mathbf{v} = \mathbf{D}\mathbf{u} + \mathbf{E}\mathbf{w} + V_0\mathbf{1}$$

- Let $\mathbf{k}_i^\top$ be the i-th row of $\mathbf{K}$
- Require that each constraint in $\mathbf{K}\mathbf{v} \leq \boldsymbol{\kappa}$ holds with probability α_i

$$\text{Prob}[\mathbf{k}_i^\top \mathbf{v} \leq \kappa_i] \geq \alpha_i, \quad i = 1, \ldots 2NT$$

- Uncertainty also renders objective function random
  - Minimize expected value

Affine policies

- Key idea: Make control action *adaptive* to uncertainty
- Linear policy

$$\mathbf{u} = \mathbf{M}\mathbf{w} + \mathbf{h}$$

- Aim is to determine $\mathbf{M}, \mathbf{h}$
- **Causality:** Control at time t depends on previous uncertainty realizations, not future ones
- **Decentralized control:** Decisions of node m depend only on uncertainty of node m
  - Does not require communication
- **Centralized control:** Decisions of node m depend on uncertainty of all nodes
  - Requires communication
- Previous constraints are linear, represented as $\mathbf{M} \in \mathcal{M}$

Objective and constraints

- Substituting $\mathbf{u} = \mathbf{M}\mathbf{w} + \mathbf{h}$ into the objective yields a **convex quadratic** in $\mathbf{M}, \mathbf{h}$

$$\mathsf{E}\left[\mathbf{u}^\top \mathbf{Q}\mathbf{u} + \mathbf{w}^\top \mathbf{R}\mathbf{w} + \mathbf{u}^\top \mathbf{S}\mathbf{w} + \mathbf{w}^\top \mathbf{S}^\top \mathbf{u}\right] = \mathsf{Tr}[\mathbf{R}\mathbf{\Sigma}] + (\bar{\mathbf{w}}^\top \mathbf{R}\bar{\mathbf{w}}) + \left\|\mathbf{Q}^{1/2}\mathbf{M}\mathbf{L}\right\|_F^2$$
$$+\ \bar{\mathbf{w}}^\top \mathbf{M}^\top \mathbf{Q}\mathbf{M}\bar{\mathbf{w}} + 2\mathbf{h}^\top \mathbf{Q}\mathbf{M}\bar{\mathbf{w}} + \mathbf{h}^\top \mathbf{Q}\mathbf{h} + 2\mathbf{h}^\top \mathbf{S}\bar{\mathbf{w}} + 2\mathsf{Tr}[\mathbf{M}^\top \mathbf{S}\mathbf{\Sigma}] + 2\bar{\mathbf{w}}^\top \mathbf{M}^\top \mathbf{S}\bar{\mathbf{w}}$$

- Chance constraint on voltages becomes SOCP constraint

$$\mathsf{Prob}\{\mathbf{k}_i^\top [\mathbf{D}(\mathbf{M}\mathbf{w} + \mathbf{h}) + \mathbf{E}\mathbf{w} + \mathbf{1}V_0] \leq \kappa_i\} \geq \alpha_i \iff$$
$$\mathbf{k}_i^\top (\mathbf{D}\mathbf{M} + \mathbf{E})\bar{\mathbf{w}} + \Phi^{-1}(\alpha_i)\left\|\mathbf{L}^\top (\mathbf{D}\mathbf{M} + \mathbf{E})^\top \mathbf{k}_i\right\|_2 \leq \kappa_i - \mathbf{k}_i^\top (\mathbf{D}\mathbf{h} + \mathbf{1}V_0)$$

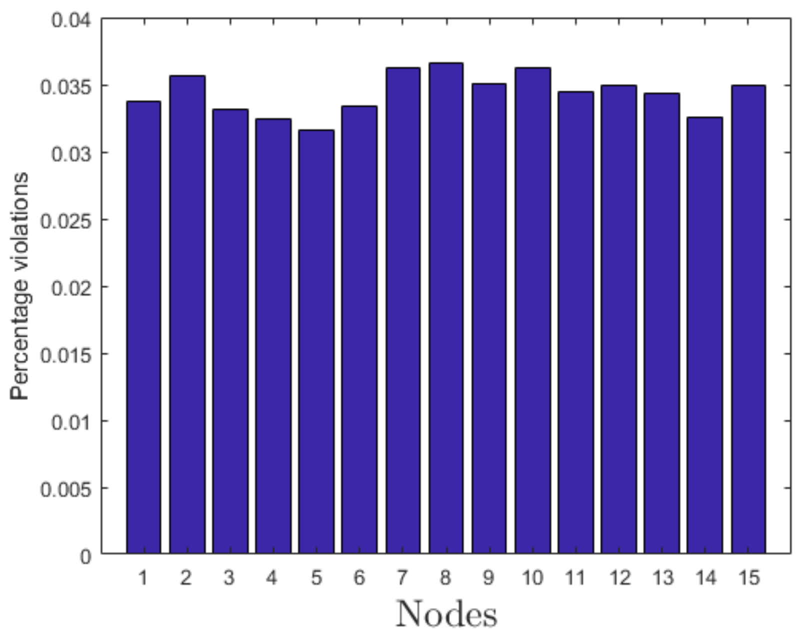
- **Complication:** Affine policy renders the left-hand sides of hard constraints (e.g., state and input bounds $\mathbf{F}\mathbf{x} \leq \phi, \mathbf{G}\mathbf{u} \leq \gamma$) random
- **Solution:** Enforce these as chance constraints, but with tighter probability specifications → SOCP constraints [Oldewurtel et al. '08-'10]

Chance constrained problem as SOCP

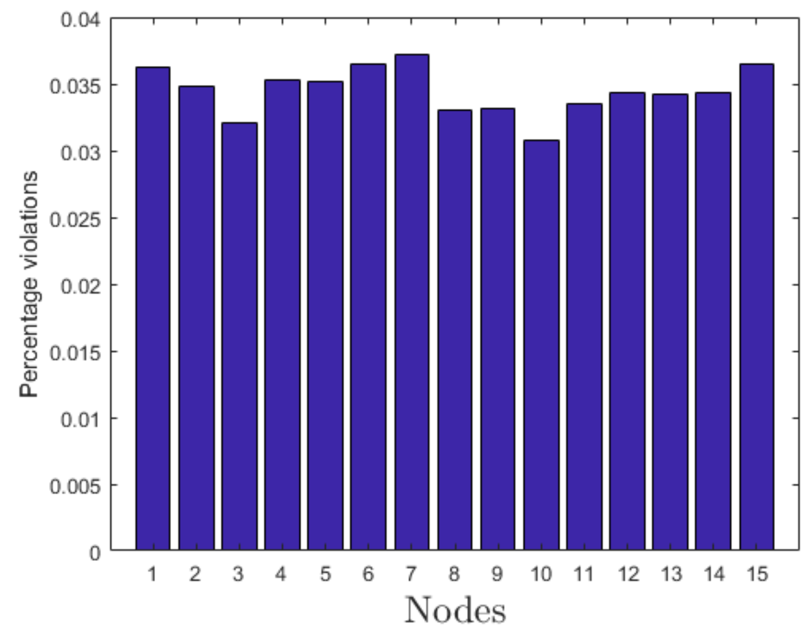
$$
\begin{aligned}
\min_{\mathbf{M},\mathbf{h}} \quad & \mathsf{Tr}[\mathbf{R}\boldsymbol{\Sigma}] + (\bar{\mathbf{w}}^\top \mathbf{R}\bar{\mathbf{w}}) + \left\| \mathbf{Q}^{1/2}\mathbf{M}\mathbf{L} \right\|_F^2 + \bar{\mathbf{w}}^\top \mathbf{M}^\top \mathbf{Q}\mathbf{M}\bar{\mathbf{w}} + 2\mathbf{h}^\top \mathbf{Q}\mathbf{M}\bar{\mathbf{w}} + \mathbf{h}^\top \mathbf{Q}\mathbf{h} \\
& + 2\mathbf{h}^\top \mathbf{S}\bar{\mathbf{w}} + 2\mathsf{Tr}[\mathbf{M}^\top \mathbf{S}\boldsymbol{\Sigma}] + 2\bar{\mathbf{w}}^\top \mathbf{M}^\top \mathbf{S}\bar{\mathbf{w}} \\
& \text{subj. to} \\
& (\boldsymbol{\xi}_{1i}^\top + \boldsymbol{\xi}_{2i}^\top \mathbf{M})\bar{\mathbf{w}} + \Phi^{-1}(\alpha_{\boldsymbol{\psi}_i}) \left\| \mathbf{L}^\top (\boldsymbol{\xi}_{1i} + \mathbf{M}^\top \boldsymbol{\xi}_{2i}) \right\|_2 \leq \psi_i - \xi_{2i}^\top \mathbf{h}, \; i = 1, \ldots, \ell NT \\
& \mathbf{z}_i^\top \mathbf{M}\bar{\mathbf{w}} + \Phi^{-1}(\alpha_{\boldsymbol{\zeta}_i}) \left\| \mathbf{L}^\top \mathbf{M}^\top \mathbf{z}_i \right\|_2 \leq \zeta_i - \mathbf{z}_i^\top \mathbf{h}, \; i = 1, \ldots, \ell NT \\
& \mathbf{f}_i^\top \mathbf{B}\mathbf{M}\bar{\mathbf{w}} + \Phi^{-1}(\alpha_{\boldsymbol{\phi}_i}) \left\| \mathbf{L}^\top \mathbf{M}^\top \mathbf{B}^\top \mathbf{f}_i \right\|_2 \leq \phi_i - \mathbf{f}_i^\top (\mathbf{A}\mathbf{x}(1) + \mathbf{B}\mathbf{h}), \; i = 1, \ldots, N(2T+1) \\
& \mathbf{g}_i^\top \mathbf{M}\bar{\mathbf{w}} + \Phi^{-1}(\alpha_{\boldsymbol{\gamma}_i}) \left\| \mathbf{L}^\top \mathbf{M}^\top \mathbf{g}_i \right\|_2 \leq \gamma_i - \mathbf{g}_i^\top \mathbf{h}, \; i = 1, \ldots, 2NT \\
& \mathbf{k}_i^\top (\mathbf{D}\mathbf{M} + \mathbf{E})\bar{\mathbf{w}} + \Phi^{-1}(\alpha_i) \left\| \mathbf{L}^\top (\mathbf{D}\mathbf{M} + \mathbf{E})^\top \mathbf{k}_i \right\|_2 \leq \kappa_i - \mathbf{k}_i^\top (\mathbf{D}\mathbf{h} + \mathbf{1}V_0), \; i = 1, \ldots, 2NT \\
& \mathbf{M} \in \mathcal{M}.
\end{aligned}
$$

Numerical tests

- Network with $N = 15$ nodes; Avg. PV profile from NREL data (Apr. 4, 2006); $\delta = 5 \text{ min}$
- No storage here (included in the paper)
- V_{min} = 0.94, V_{max}=1.06
- Probability spec. for voltage violation 85%; Probability spec. for all other constraints 95%
- 10,000 scenarios of PV generation drawn for validation
- Figures show % of scenarios with $|q_{m,t}^{PV}| > \sqrt{(S_{m,\max}^{PV})^2 - (p_{m,t}^{PV})^2}$



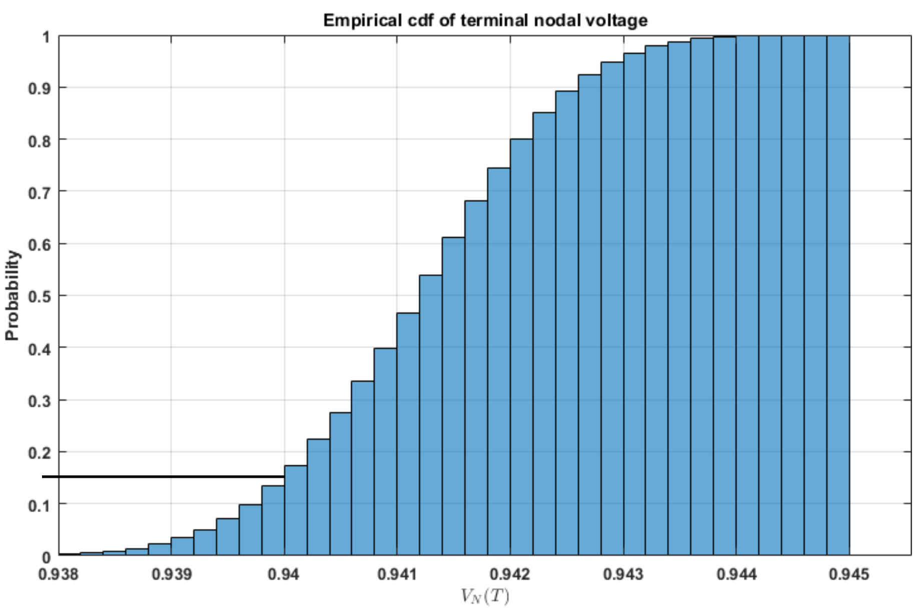
Decentralized



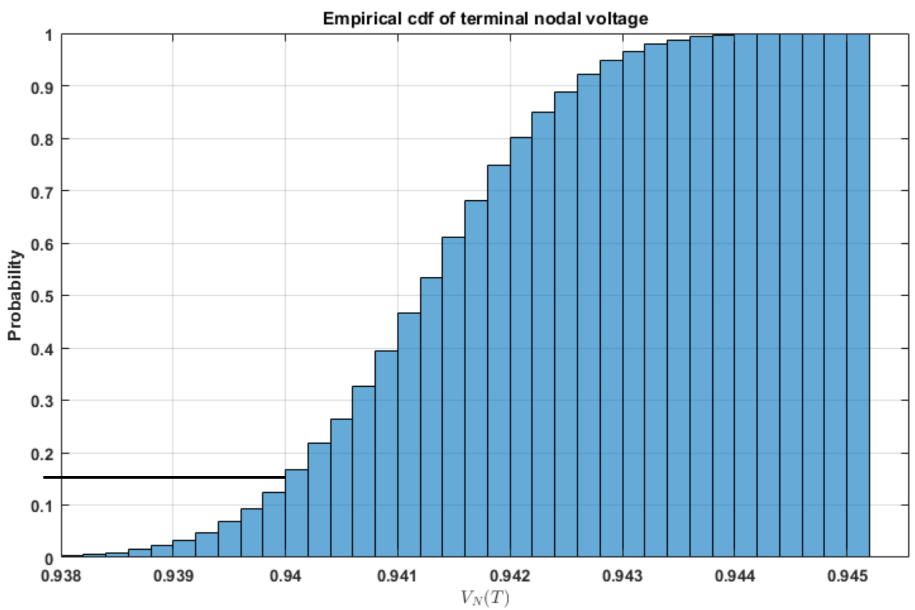
Centralized

Numerical tests

- $V_{min} = 0.94, V_{max}=1.06$
- Probability spec. for voltage violation 85%; probability spec. for all other constraints 95%
- 10,000 scenarios of PV gen. drawn for validation
- $q^{PV}_{m,t}$ projected back to feasible set; resulting voltages computed from DistFlow
- Centralized design slightly pushes the voltage CDF to the right, decreases objective
- Probability specification satisfied by empirical CDF (15% of scenarios are below V_{min})



Decentralized



Centralized

Summary

- Chance constrained optimization of distributed generation and storage
  - Reactive power from PV inverters
  - Storage charge/discharge and reactive power support
  - Affine policy for decision variables
  - Overall problem is SOCP

- Future directions
  - Tests on tree networks
  - Scaling of the approach to larger networks, custom algorithms

Thank you!

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