Optimal Local Thresholds for Distributed Detection in Energy Harvesting Wireless Sensor Networks

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Outline

- Introduction
- System Model and problem statement
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The designs of wireless sensor networks to perform the task of distributed detection are often based on the conventional battery-powered sensors, leading into designs with a short lifetime, due to battery depletion.

Energy harvesting, which can collect energy from renewable resources of environment (e.g., solar, wind, and geothermal energy) promises a self-sustainable system with a lifetime.
System Model

\[ H_0 : A \text{ is present} \]
\[ H_1 : A \text{ is absent} \]

\[ x_1, x_2, \ldots, x_K \]
\[ \Omega_1, \Omega_2, \ldots, \Omega_K \]
\[ b_1, b_2, \ldots, b_K \]

subject to additive \( w_k \) and multiplicative \( b_k \) observation noises

\[ \text{subject to addi5ve } w_k \text{ and multiplicative } b_k \text{ observation noises} \]

Figure 1: Our System model during one observation period.
Let $x_k$ denote the local observation at sensor $k$:

$$ x_k = \begin{cases} 
  g_k A + w_k & \mathcal{H}_1 \\
  w_k & \mathcal{H}_0 
\end{cases} \quad (1) $$

- $A$ is a known scalar signal
- $w_k \sim \mathcal{N}(0, \sigma^2_{w_k}) \rightarrow$ Additive noise
- $g_k \sim \mathcal{N}(0, \gamma_{g_k}) \rightarrow$ Multiplicative noise
- All observation noises are independent over time and among $K$ sensors.
System Model

During each observation period, sensor $k$ takes $N$ samples of $x_k$ to measure the received signal energy and applies an energy detector to make a binary decision, i.e., sensor $k$ decides whether or not signal $A$ is present.

$$\Lambda_k = \frac{1}{N} \sum_{n=1}^{N} |x_{k,n}|^2 \geq \begin{cases} d_k = 1 & \theta_k \\ d_k = 0 & \end{cases}$$

(2)

- $P_{f_k} = \text{Pr}(\Lambda_k > \theta_k | H_0) = \frac{\Gamma \left( N/2, \frac{N\theta_k}{\sigma^2_{w_k}} \right)}{\Gamma(N/2)}$
- $P_{d_k} = \text{Pr}(\Lambda_k > \theta_k | H_1) = Q_{N/2} \left( \frac{\sqrt{\eta_k}}{\sigma_{w_k}}, \frac{\sqrt{N\theta_k}}{\sigma_{w_k}} \right)$

*Our goal is optimize the local decision threshold $\theta_k$*
Assumptions:

- Each sensor is able to harvest energy from the environment and stores it in a battery with the capacity $K$ units of energy.
- The sensors communicate with the FC through orthogonal fading channels with channel gains $|h_k|$’s with parameters $\gamma h_k$.
- The sensors employ on-off keying signaling.
- We use the channel-inversion power, the number of energy units spent to convey a decision is inversely proportional to $|h_k|$.
- To avoid the battery depletion when $|h_k|$ is too small, we impose an extra constraint for channel quality.
Let $u_{k,t}$ represent the sensor output corresponding to the observation period $t$.

$$u_{k,t} = \begin{cases} 
\left\lceil \frac{\lambda}{|h_k|} \right\rceil & \Lambda_k > \theta_k, \ b_{k,t} > \left\lceil \frac{\lambda}{|h_k|} \right\rceil, \ |h_k|^2 > \zeta_k \\
0 & \text{Otherwise}
\end{cases}$$

(3)

- $b_{k,t}$ denote the battery state of sensor $k$
- $|h_k|$ is channel gain
- $\zeta_k$ is threshold of the channel quality
- $\lambda$ is a power regulation constant
We model $b_{k,t}$ in (3) as the following

$$b_{k,t} = \min \{ b_{k,t-1} - \left\lfloor \frac{\lambda}{|h_k|} \right\rfloor l_{u_{k,t-1}} + \Omega_{k,t}, K \}$$  \hspace{1cm} (4)

- $\Omega_{k,t} \in \{0, 1\}$ indicates units of harvesting energy and it is a Bernoulli random variable, with $\Pr(\Omega_{k,t} = 1) = p_e$
- $l_{u_{k,t-1}} = \begin{cases} 1 & u_{k,t-1} > 0 \\ 0 & \text{Otherwise} \end{cases}$
Assuming $b_k$ in (4) is a stationary random process, one can compute the CDF and the pmf of $b_k$ in terms of $\mathcal{K}, p_e, \gamma h_k$. Further, we use pmf of $b_k$ for our numerical results.

**Figure 2:** (a) CDF of $b_k$ for $\mathcal{K}=20$ and $p_e=0.5, 0.75, 0.82$, (b) pmf of $b_k$ for $\mathcal{K}=50$ and $p_e=0.8$. 
We consider two detection performance metrics to find the optimal $\theta_k$’s:

- The detection probability at the FC, assuming that the FC utilizes the optimal fusion rule based on Neyman-Pearson optimality criterion.

- The KL distance between the two distributions of the received signals at the FC conditioned on hypothesis $\mathcal{H}_0, \mathcal{H}_1$
The received signal at the FC from sensor $k$ is $y_k = h_k u_k + n_k$, where the additive communication channel noise $n_k \sim \mathcal{N}(0, \sigma_{n_k}^2)$. The likelihood ratio at the FC is

$$\Delta_{\text{LRT}} = \sum_{k=1}^{K} \log \left( \frac{\sum_{u_k} f(y_k | u_k, \mathcal{H}_1) \Pr (u_k | \mathcal{H}_1)}{\sum_{u_k} f(y_k | u_k, \mathcal{H}_0) \Pr (u_k | \mathcal{H}_0)} \right)$$

(5)

Given $u_k$, $y_k$ is Gaussian, i.e., $y_k | u_k=0 \sim \mathcal{N}(0, \sigma_{n_k}^2)$ and $y_k | u_k = \left\lfloor \frac{\lambda}{|h_k|} \right\rfloor \sim \mathcal{N}\left( \left\lfloor \frac{\lambda}{|h_k|} \right\rfloor h_k, \sigma_{n_k}^2 \right)$. 
Optimal LRT Fusion Rule and $P_D, P_F$ Expressions

The probabilities $\Pr(u_k|\mathcal{H}_1)$, $\Pr(u_k|\mathcal{H}_0)$ in (5) are

- $\Pr(u_k = \lceil \frac{\lambda}{|h_k|} \rceil | \mathcal{H}_1) = P_{d_k} \rho_k q_k = \alpha_k$
- $\Pr(u_k = \lceil \frac{\lambda}{|h_k|} \rceil | \mathcal{H}_0) = P_{f_k} \rho_k q_k = \beta_k$

where $\rho_k = \Pr(b_k > \lceil \frac{\lambda}{|h_k|} \rceil)$ and $q_k = \Pr(|h_k|^2 > \zeta_k)$.

Given a threshold $\tau$, the optimal likelihood ratio test (LRT) is

$$\Delta_{\text{LRT}} \geq \frac{\mathcal{H}_1}{\mathcal{H}_0} \tau.$$ The $P_F, P_D$ at the FC

$$P_F = \Pr(\Delta_{\text{LRT}} > \tau | \mathcal{H}_0) = Q\left( \frac{\tau - \mu_\Delta |\mathcal{H}_0|}{\sigma_\Delta |\mathcal{H}_0|} \right) \quad (6)$$

$$P_D = \Pr(\Delta_{\text{LRT}} > \tau | \mathcal{H}_1) = Q\left( \frac{Q^{-1}(a)\sigma_\Delta |\mathcal{H}_0| + \mu_\Delta |\mathcal{H}_0| - \mu_\Delta |\mathcal{H}_1|}{\sigma_\Delta |\mathcal{H}_1|} \right) \quad (7)$$
Kullback-Leibler distance (KL) between the two distributions of the received signals at the FC

\[ KL_k = \int_{y_k} f(y_k | \mathcal{H}_1) \log \left( \frac{f(y_k | \mathcal{H}_1)}{f(y_k | \mathcal{H}_0)} \right) dy_k \]  

(8)

One can approximate \( KL_k \) in (8) by the KL distance of two Gaussian distributions

\[ KL_k \approx \frac{1}{2} \log \left( \frac{\sigma^2_{y_k | \mathcal{H}_0}}{\sigma^2_{y_k | \mathcal{H}_1}} \right) + \frac{\sigma^2_{y_k | \mathcal{H}_1} - \sigma^2_{y_k | \mathcal{H}_0} + (\mu_{y_k | \mathcal{H}_1} - \mu_{y_k | \mathcal{H}_0})^2}{2\sigma^2_{y_k | \mathcal{H}_0}} \]  

(9)
In this section, we consider:

- **Scheme I**: Numerically find $\theta_k$’s which maximize $P_D$ in (7) $\rightarrow$ $K$-dimensional search is required $\rightarrow$ computational complexity!

- **Scheme II**: Finding $\theta_k$’s which maximize $KL_{tot} = \sum_{k=1}^{K} KL_k$, using the $KL_k$ approximation in (9) $\rightarrow$ Only one dimensional search $\rightarrow$ computationally efficient.

- **Special case**: Assume all sensors employ the same local threshold $\theta_k = \theta$ and compare schemes I and II.

We then compare $P_D$ evaluated at the $\theta_k$’s obtained from mentioned schemes.
Simulation results

Figure 3: (a) $P_D$ vs. $P_F$
(b) $P_D$ vs. $\kappa$
Conclusion

- We studied a distributed detection problem in a wireless network with $K$ heterogeneous energy harvesting sensors and investigated the optimal local decision thresholds for given transmission and battery state models.

- Our numerical results indicate that the thresholds obtained from maximizing the KL distance are near-optimal and computationally very efficient, as it requires only $K$ one-dimensional searches, as opposed to a $K$-dimensional search required to find the thresholds that maximize the detection probability.

- The performance gap between each scheme and its corresponding special case indicates that when sensors are heterogeneous, it is advantageous to use different local thresholds according to sensors’ statistics.
Questions?