COMPRESSED TRAINING ADAPTIVE EQUALIZATION
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What is Compressed Training Adaptive Equalization?
- An adaptive equalization framework aiming to reduce the number of training symbols in a communication packet. The equalizer coefficients are trained by exploiting:
  - Training symbols,
  - Magnitude-boundedness property of digital communication constellations.

Highlights of the Framework
- Direct link with Compressed Sensing,
- Reduce Training Length,
- Desirable:
  - Minimum Training Length & log(Channel Spread)
- Algorithms Based on Convex Settings,
- Do NOT Make Sparse Channel Assumption.

Equalization Setup
- We assume the standard Fractionally-Spaced Equalization Setup.

The Proposed Framework
Noiseless/High SNR Case:
- The proposed optimization setting:
  
  \[ \text{minimize} \quad \|a\|_\infty \quad \text{subject to} \quad y_F w = s_F. \] 

Connection to Compressed Sensing
- For the noiseless scenario, \( a \) can be written as:
  \[ a = g_0 a_0 + g_1 a_1 + \cdots + g_L a_{L-1} \]
- For sufficiently long data packet and BFSK constellation,
  \[ \|a\|_\infty \leq 1. \]
- The corresponding dual optimization setting:
  \[ \text{minimize} \quad \|\beta\|_1 \quad \text{subject to} \quad \beta = y_F^T. \]

Connection to Compressed Sensing
- We observe Setting I is equivalent to Sparse Reconstruction Problem if we consider:
  - \( y_F \) as the observation vector,
  - \( S \) as the measurement matrix and,
  - \( g \) as the one-sparse vector to be reconstructed.

Analysis of the Proposed Approach
- The mutual coherence of the matrix \( S \in \mathbb{R}^{n \times L} \) is defined as \( \mu(S) = \max_{i \neq j} \frac{|S_{ij}|}{\sqrt{\|S_i\|_2 \cdot \|S_j\|_2}} \)
- Theorem 2: Let \( S \in \mathbb{R}^{n \times L} \) be full rank with \( L \geq L_0 \).
  If the system of linear equations \( Sg = y \) has a solution \( g \), which obeys
  \[ \|g\|_0 < 0.5 \left( 1 + \mu(S)^{-1} \right), \]
  then it is the unique solution for the optimization problem in Setting I.

CONCLUSION
- We introduced convex optimization based Adaptive Equalization Framework that reduces training data to \( O(\log(\text{Channel-Spread})) \) as opposed to \( O(\text{Channel-Spread}) \).
- A duality based link between the proposed approach and compressed sensing is established.

REFERENCES

Noisy Case Communication Example
- SNR is chosen as 25dB,
- Compared with least squares and the blind algorithm in [3]:

- Channel Length=15 and Equalizer Length=20,
- Success probability is defined as \( |g - e_{1:2}^\top| \leq 10^{-5}, \)
- Comparison with the algorithm in [4](CMA+LS),
- Empirical probability vs. the bounds and Mean Square Error Performance.

Noisy Case Communication Example

Data Training Data
- The optimization setting factoring existence of noise:
  \[ \text{minimize} \quad \|y_F w - y_F^T\|_2 \quad \text{subject to} \quad \|a\|_\infty \leq \gamma \]

Setting I: \( \ell_1 \)-LASSO
- \( \gamma \) represents the knowledge about the symbol boundedness.
- Alternative convex optimization setting for the noisy case:
  \[ \text{minimize} \quad \|y_F w - y_F^T\|_2 + \lambda \|a\|_\infty \]

Perfect Equalization Condition: \( g_0 = \delta_{L-1}. \)

\( \lambda \) is the regularization parameter.

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