Subspace-based Adaptive Widely Linear Blind Channel Estimation for Constrained Minimum Variance CDMA Receiver

Nuan Song[1], Vimal Radhakrishnan[2], Rodrigo C. de Lamare[3], Martin Haardt[2]

[1]Shanghai Radio Equipment Research Institute, Shanghai, China
[2]Communication Research Laboratory, Ilmenau University of Technology, Germany
[3]CETUC/PUC-Rio, Brazil, and Communications Research Group, University of York, U.K.

Introduction

- **Blind channel estimation** is attractive since no transmission of training sequences is required ⇒ saves channel bandwidth
- Conventional subspace-based methods rely on Singular Value Decomposition (SVD)
  – high computational complexity for a system with a large processing gain
  – erroneous rank estimation ⇒ a drastic performance degradation
- Iterative power method [1]:
  + eliminates SVD
  + reduces computational complexity

- **Channel estimation** requires second-order statistics of the data $\boldsymbol{r} \in \mathbb{C}^M$
  - Usually assume $\boldsymbol{r}$ is second-order circular ⇒ covariance matrix $\boldsymbol{R} = \mathbb{E}\{\boldsymbol{r}\boldsymbol{r}^H\}$ ($\boldsymbol{r}$: zero-mean)
  - However when $\boldsymbol{r}$ is **non-circular** ⇒ pseudo-covariance matrix $\check{\boldsymbol{R}} = \mathbb{E}\{\boldsymbol{r}\boldsymbol{r}^T\} \neq \boldsymbol{0}$
- **Widely Linear (WL)** processing
  - improves performance by ***fully*** exploiting the ***non-circularity*** of $\boldsymbol{r}$ and taking advantage of the second-order statistics $\boldsymbol{R}$ & $\check{\boldsymbol{R}}$ [2]
  - constructs ***virtual*** measurements ⇒ saves the hardware resources VERSUS oversampling or using several sensors

Subspace-based Widely Linear (WL) blind channel estimation

- Existing work is based on SVD [3, 4]
  - computational complexity is much higher in the WL case
- ★ Propose a subspace-based WL blind channel estimation scheme based on the **iterative power method** for the **WL Constrained Minimum Variance (WL-CMV)** CDMA receiver for **non-circular** signals

System Model and Receiver

The received vector: $\boldsymbol{r}(i) = \sqrt{E_1} b_1(i) \boldsymbol{C}_1 \boldsymbol{h}_1 + \boldsymbol{v}(i) + \boldsymbol{\eta}(i) + \boldsymbol{n}(i) \quad \in \mathbb{C}^M$

- $b_1(i) \in \{\pm 1\}$: the i-th Binary Phase Shift Keying (BPSK) symbol for the desired user 1 with unit variance
- $\boldsymbol{C}_1 \in \mathbb{R}^{M \times L}$ (Toeplitz matrix): code matrix of the desired user
- $\boldsymbol{h}_1 \in \mathbb{C}^L$: complex channel vector with normalization
- $\boldsymbol{n}(i) \in \mathbb{C}^{M \times 1}$: AWGN with power spectrum density N_0
- $\boldsymbol{v}(i), \boldsymbol{\eta}(i)$: Multi-User Interference (MUI) part and Intra-/Inter-Symbol Interference (ISI) part

Linear receiver: $\boldsymbol{r} \in \mathbb{C}^M$, $y = \boldsymbol{w}^H \boldsymbol{r}$

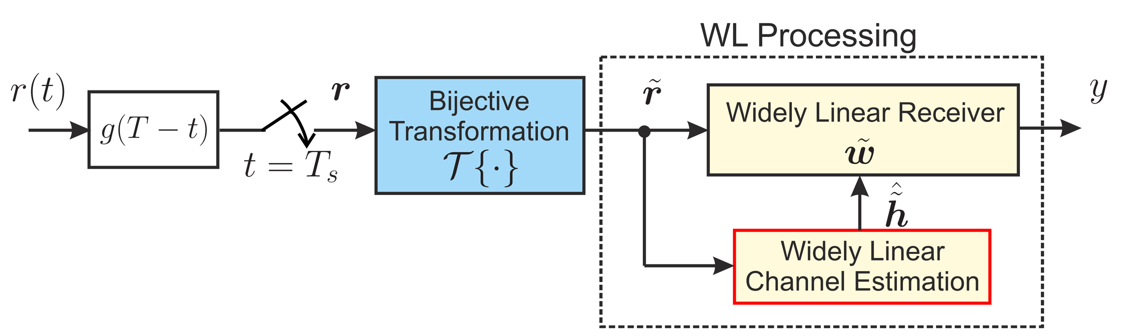
$[\cdot] \rightarrow \mathcal{T} \rightarrow [\tilde{\cdot}]$ Bijective Transformation

WL receiver, **augmented**: $\tilde{\boldsymbol{r}} = [\boldsymbol{r}^T, \boldsymbol{r}^H]^T / \sqrt{2} \in \mathbb{C}^{2M}$, $y = \tilde{\boldsymbol{w}}^H \tilde{\boldsymbol{r}}$

2nd-order statistics: $\boldsymbol{R} \in \mathbb{C}^{M \times M}$

augmented covariance matrix

$$\tilde{\boldsymbol{R}} = \mathbb{E}\{\tilde{\boldsymbol{r}}(i)\tilde{\boldsymbol{r}}^H(i)\} = \frac{1}{2}\begin{bmatrix} \boldsymbol{R} & \check{\boldsymbol{R}} \\ \check{\boldsymbol{R}}^* & \boldsymbol{R}^* \end{bmatrix} \in \mathbb{C}^{2M \times 2M} \quad (1)$$



WL-CMV receiver:

$$\tilde{\boldsymbol{w}} = \frac{\tilde{\boldsymbol{R}}^{-1}\tilde{\boldsymbol{C}}_1\tilde{\boldsymbol{h}}_1}{\tilde{\boldsymbol{h}}_1^H\tilde{\boldsymbol{C}}_1^H\tilde{\boldsymbol{R}}^{-1}\tilde{\boldsymbol{C}}_1\tilde{\boldsymbol{h}}_1}, \quad \tilde{\boldsymbol{C}}_1 = \begin{bmatrix} C_1 & 0 \\ 0 & C_1 \end{bmatrix} / \sqrt{2} \quad (2)$$

need channel estimation $\hat{\tilde{\boldsymbol{h}}}_1 \rightarrow \tilde{\boldsymbol{h}}_1$

Subspace-based WL Channel Estimation

$$\tilde{\boldsymbol{R}} = [\tilde{\boldsymbol{U}}_s \; \tilde{\boldsymbol{U}}_n] \begin{bmatrix} \tilde{\boldsymbol{\Lambda}}_s + \frac{N_0}{2}\boldsymbol{I}_K & \boldsymbol{0} \\ \boldsymbol{0} & \frac{N_0}{2}\boldsymbol{I}_{2M-K} \end{bmatrix} [\tilde{\boldsymbol{U}}_s \; \tilde{\boldsymbol{U}}_n]^H$$

- $\tilde{\boldsymbol{\Lambda}}_s = \mathrm{diag}\{\lambda_1, \cdots, \lambda_K\}$: singular values
- $\tilde{\boldsymbol{U}}_s \in$ augmented signal subspace
- $\tilde{\boldsymbol{U}}_n \in$ augmented noise subspace
- Orthogonality ⇒ $\tilde{\boldsymbol{U}}_n \tilde{\boldsymbol{C}}_1 \tilde{\boldsymbol{h}}_1 = \boldsymbol{0}$

Optimization problem for WL subspace-based channel estimation:

$$\hat{\tilde{\boldsymbol{h}}}_1 = \arg\min_{\tilde{\boldsymbol{h}}_1} \tilde{\boldsymbol{h}}_1^H \tilde{\boldsymbol{C}}_1^H \tilde{\boldsymbol{U}}_n \tilde{\boldsymbol{U}}_n^H \tilde{\boldsymbol{C}}_1 \tilde{\boldsymbol{h}}_1, \quad \|\tilde{\boldsymbol{h}}_1\| = 1$$

the power of the R concept [7]

Simplified optimization problem:

$$\hat{\tilde{\boldsymbol{h}}}_1 = \arg\min_{\tilde{\boldsymbol{h}}_1} \tilde{\boldsymbol{h}}_1^H \boldsymbol{W} \tilde{\boldsymbol{h}}_1, \quad \|\tilde{\boldsymbol{h}}_1\| = 1 \;\text{ with }\; \boldsymbol{W} = \tilde{\boldsymbol{C}}_1^H \tilde{\boldsymbol{R}}^{-m} \tilde{\boldsymbol{C}}_1 \in \mathbb{C}^{L \times L}, \; m = 1, 2, \cdots \quad (3)$$

$\hat{\tilde{\boldsymbol{h}}}_1$: singular vector → the smallest singular value of $\boldsymbol{W}$

WL iterative power method:

$$\tilde{\boldsymbol{h}}_1(i) = \frac{(\boldsymbol{I}_{2L} - \beta\boldsymbol{W})\tilde{\boldsymbol{h}}_1(i-1)}{\left\|(\boldsymbol{I}_{2L} - \beta\boldsymbol{W})\tilde{\boldsymbol{h}}_1(i-1)\right\|}, \quad \beta = 1/\mathrm{tr}\{\boldsymbol{W}\} \quad (4)$$

- $\tilde{\boldsymbol{h}}_1(i) \rightarrow \tilde{\boldsymbol{h}}_1$ with sign ambiguity

★ with (4): SVD is avoided to solve (3) ⇒ simplifies the implementation
★ **WL** channel estimation: the phase ambiguity reduces to a sign ambiguity

Adaptive Algorithms

(4) should also be updated with

$$\boldsymbol{W}(i) = \tilde{\boldsymbol{C}}_1^H \tilde{\boldsymbol{R}}^{-m}(i) \tilde{\boldsymbol{C}}_1, \; m = 1, 2, 3, \quad \beta(i) = \frac{1}{\mathrm{tr}\{\boldsymbol{W}(i)\}}. \quad (5)$$

Two adaptive algorithms based on Recursive Least Squares (RLS):

- Augmented **RLS** (**A-RLS**): directly utilizes $\tilde{\boldsymbol{r}}(i)$
- Structured **RLS** (**S-RLS**): exploits block conjugate structure of $\tilde{\boldsymbol{R}}(i)$ in (1)

Aim: estimate $\tilde{\boldsymbol{h}}_1(i)$ and $\tilde{\boldsymbol{w}}(i)$ at time i

Algorithm:

1. Update $\tilde{\boldsymbol{R}}^{-1}(i) \Leftarrow$ given $\tilde{\boldsymbol{r}}(i)$
2. $\tilde{\boldsymbol{h}}_1(i) \overset{(4)(5)}{\Leftarrow} \tilde{\boldsymbol{R}}^{-1}(i) \Leftarrow$ given $m, \tilde{\boldsymbol{C}}_1$
3. Update receiver $\tilde{\boldsymbol{w}}(i) \overset{(2)}{\Leftarrow} \tilde{\boldsymbol{R}}^{-1}(i), \tilde{\boldsymbol{h}}_1(i)$

- Update $\tilde{\boldsymbol{R}}^{-1}(i)$ in **A-RLS**

$$\tilde{\boldsymbol{R}}^{-1}(i) = \lambda^{-1}\tilde{\boldsymbol{R}}^{-1}(i-1) - \lambda^{-1}\boldsymbol{k}(i)\tilde{\boldsymbol{r}}^H(i)\tilde{\boldsymbol{R}}^{-1}(i-1), \quad (6)$$

$$\boldsymbol{k}(i) = \frac{\lambda^{-1}\tilde{\boldsymbol{R}}^{-1}(i-1)\tilde{\boldsymbol{r}}(i)}{1 + \lambda^{-1}\tilde{\boldsymbol{r}}^H(i)\tilde{\boldsymbol{R}}^{-1}(i-1)\tilde{\boldsymbol{r}}(i)}, \; \lambda\text{: forgetting factor}$$

- In **S-RLS**, an efficient way to update $\tilde{\boldsymbol{R}}^{-1}(i)$ as in [5, 6]

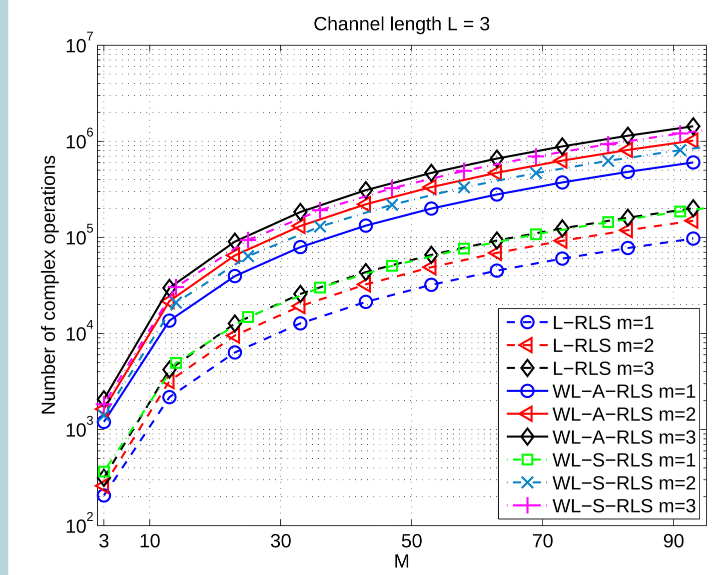
$$\tilde{\boldsymbol{R}}^{-1}(i) = \begin{bmatrix} \boldsymbol{P}(i) & \boldsymbol{Q}(i) \\ \boldsymbol{Q}^*(i) & \boldsymbol{P}^*(i) \end{bmatrix}, \quad (7)$$

★	A-RLS	S-RLS
Initialization[a]	$\tilde{\boldsymbol{R}}^{-1}(0) = \delta_a \boldsymbol{I}_{2M}$	$\boldsymbol{P}(0) = \delta_p \boldsymbol{I}_M, \boldsymbol{Q}(0) = \delta_q \boldsymbol{I}_M$
$\tilde{\boldsymbol{R}}^{-1}(i)$	by (6)	update $\boldsymbol{P}(i), \boldsymbol{Q}(i)$
$\tilde{\boldsymbol{h}}_1(i)$	by (4)	use structured $\tilde{\boldsymbol{R}}^{-1}(i)$ in (5)
Complexity	$2M$	M

[a]Scalars $\delta_a, \delta_p, \delta_q$ ⇒ numerical stability

Key advantage of **S-RLS** over **A-RLS**
- structured manner
  ⇒ more efficient and lower complexity

Complexity Analysis of Adaptive Channel Estimation Algorithms

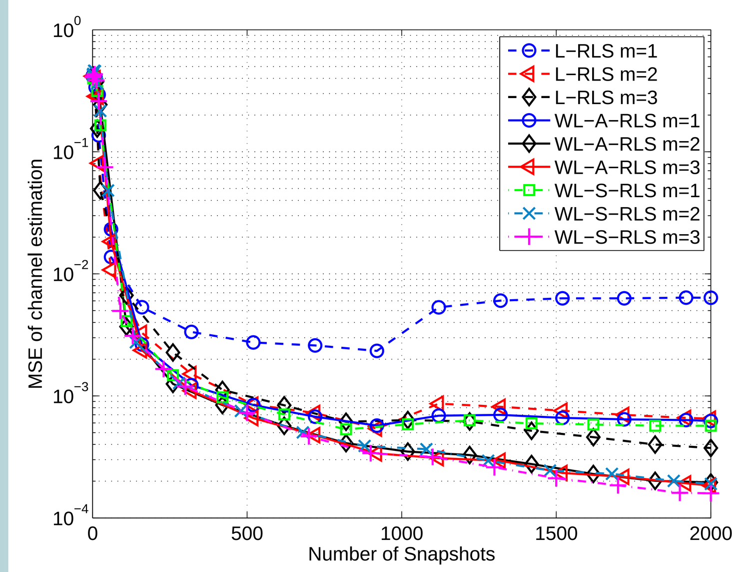


⇐Fig.: total No. of complex "+" and "×" per iteration per symbol compared to M

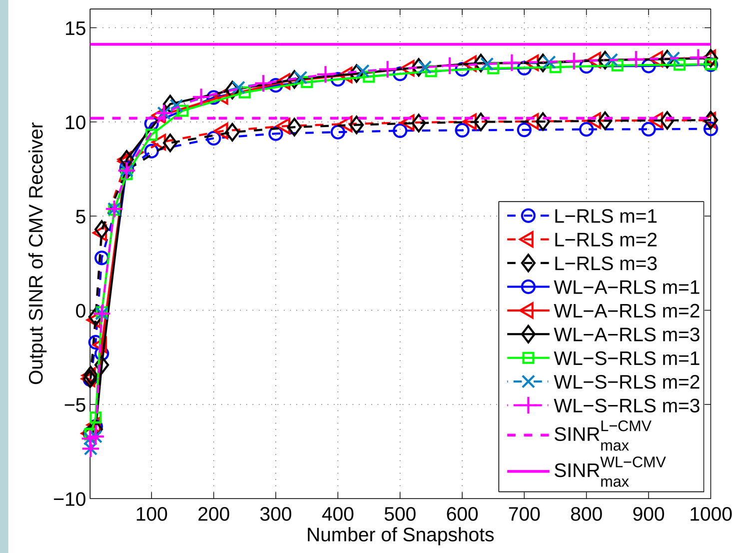
Considered Adaptive Schemes with $m = 1, 2, 3$:
- Linear RLS (L-RLS)
- Proposed WL-A-RLS
- Proposed WL-S-RLS

⇒ WL-A-RLS: high complexity
⇒ WL-S-RLS with $m = 1 \approx$ L-RLS with $m = 3$

Simulation Results



Simulation setup:
- √ CDMA system with $K = 12$ users
- √ Random spreading sequences of length $N = 32$
- √ BPSK-modulated signals (strictly non-circular)
- √ Multipath block-fading channel: length $L = 3$ and power delay profile [0,-3,-6] dB
- √ Input SNR = 12 dB
- √ Dynamic case: at bit 1000, 6 users with 10 dB more power enter the channel



- Channel estimation **MSE** in dynamic case
  - **WL** > **L** and robust in the dynamic case
  - $m = 2$ is sufficient for **WL**
  - **WL-S-RLS** slightly better than **WL-A-RLS** but with much lower complexity
- **Output SINR** for WL-CMV receiver
  - **WL** has ≥ 3 dB gain over **L**
  - **WL** channel estimation $\overset{\text{directly applied to}}{\Longrightarrow}$ blind **WL** receivers

Conclusions

- **Propose a subspace-based WL blind channel estimation based on iterative power technique**
  - **non-circular** signals ⇒ **WL** processing
  - Completely avoids computing SVD
- **Adaptive algorithms** for WL-CMV receiver: **WL-A-RLS** and **WL-S-RLS**
  - **WL-S-RLS** applies the structured property of $\tilde{\boldsymbol{R}}$
    ⇒ a faster convergence and lower complexity than **WL-A-RLS**

Selected References

[1] X. G. Doukopoulos and G. V. Moustakides, "Adaptive power techniques for blind channel estimation in CDMA systems," *IEEE Transactions on Signal Processing*, vol. 53, no. 3, pp. 1110-1120, Mar. 2005.

[2] P. J. Schreier and L. L. Scharf, "Second-order analysis of improper complex random vectors and processes," *IEEE Transactions on Signal Processing*, vol. 51, no. 3, pp. 714-725, March 2003.

[3] A. S. Cacciapuoti, G. Gelli, L. Paura, and F. Verde, "Widely Linear Versus Linear Blind Multiuser Detection With Subspace-Based Channel Estimation: Finite Sample-Size Effects," *IEEE Transactions on Signal Processing*, vol. 57, no. 4, pp. 1426-1443, Dec. 2008.

[4] S. Abdallah and I. N. Psaromiligkos, "Widely Linear versus Conventional Subspace-Based Estimation of SIMO Flat-Fading Channels: Mean Squared Error Analysis," *IEEE Transactions on Signal Processing*, vol. 60, no. 3, pp. 1307-1318, Mar. 2012.

[5] S. C. Douglas, "Widely-linear recursive least-squares algorithm for adaptive beamforming," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing*, Taipei, Taiwan, April 2009.

[6] N. Song, W. U. Alokozai, R. C. de Lamare, and M. Haardt, "Adaptive Widely Linear Reduced-Rank Beamforming Based on Joint Iterative Optimization," *IEEE Signal Processing Letters*, vol. 21, no. 3, pp. 265-269, Mar. 2014.

[7] Z. Xu, P. Liu, and X. Wang, "Blind multiuser detection: From MOE to subspace methods," *IEEE Transactions on Signal Processing*, vol. 52, no. 2, pp. 510-524, Feb. 2004.