ABSTRACT

Present methods of analysing functional networks in brain during task-conditions mainly include concatenation followed by temporal correlation. We employ Markov Chain Monte Carlo methods, namely Metropolis within Gibbs sampling, on a stochastic model to infer dynamic functional connectivity. By using a Bayesian probabilistic framework, distributional estimates are obtained as opposed to point estimates, and the uncertainty of the existence of such links is accounted for. The methodology is applied to fMRI data from a finger opposition paradigm with task and fixation conditions, investigating the dynamics of the well characterised somato-motor network while using the visual network as a control case.

MODEL

The BOLD signal is modelled such that the change in the signal value is the result of a summation of forces, which are dependent on BOLD signals from other nodes:

\[ \Delta x_{t} = \sum_{j=1}^{N} S_{ij} \varphi_{ij}(x_{j\tau_{j}} - y_{j\tau_{j}}) + \epsilon_{t} \Delta t + \sigma_{t}^{2} \eta_{t} \]

where \( x_{j} \) is the BOLD signal at node \( j \) at time \( t \), \( \varphi_{ij} \) is the interaction parameter between nodes \( i \) and \( j \), \( S_{ij} \) is an indicator variable to specify the existence of a link, \( \eta_{t} \) represents Brownian motion, \( y_{j\tau} \) is a mean-reverting term, and \( N \) is the total number of nodes.

The system over all nodes can be described as a linear stochastic differential equation of the form:

\[ \Delta X_{t} = AX_{t} + BW_{t}, \]

and the transition density is given by the following Normal distribution:

\[ p(X_{t}|X_{t-1}, S, \Phi) = N(X_{t}|F(S, \Phi)X_{t-1}, Q), \]

where \( F \) and \( Q \) can be computed. Similarly, if the observation has random, additive noise, then the observation density is given by:

\[ p(Z_{t}|X_{t}) = N(Z_{t}|X_{t}, \sigma_{z}^{2}) \]

ALGORITHM

An MCMC algorithm is used to infer the networks in different task conditions via sampling from the joint distribution \( p(S, \Phi | Z_{1:T}) \) without the need to sample the BOLD signals \( X_{1:T} \) as these are marginalised out. We would like to sample each \( \varphi_{ij} \) from its full conditional distribution:

\[ p(\varphi_{ij}|S, \varphi_{ui\neq ij}, Z_{1:T}) \]

As it is difficult to sample directly from this distribution, Metropolis within Gibbs sampling is employed in order to make inference. A proposal is made and accepted with probability given by the ratio:

\[ K = \frac{p(Z_{1:T}|S, \varphi_{ui\neq ij}, \varphi_{ij})}{p(Z_{1:T}|S, \varphi_{ui\neq ij}, \varphi_{ij}^{(n-1)})} \]

The observation likelihood, \( p(Z_{1:T}|S, \Phi) \), can be found from Kalman filtering such that:

\[ p(Z_{1:T}|S, \Phi) = p(Z_{1}) \prod_{t=2}^{T} p(Z_{t}|Z_{t-1}, S, \Phi) \]

This distribution is an integration over the hidden state \( X_{1:T} \) since:

\[ p(Z_{1:T}|S, \Phi) = p(Z_{1}|S, \Phi) \prod_{t=2}^{T} p(Z_{t}|X_{t-1}, S, \Phi) \]

and

\[ p(Z_{t}|X_{t-1}, S, \Phi) = \int p(Z_{t}|X_{t})p(X_{t}|X_{t-1}, S, \Phi)dx_{t} \]

As the indicator variables can only take one of two possible values, the probability of both possibilities can be calculated in a Gibbs Sampling framework. The posterior over \( S_{ij} \in \{0,1\} \) is given by:

\[ p(S_{ij}|Z_{1:T}, \Phi, S_{ui\neq ij}) = p(S_{ij}|Z_{1:T}, \Phi, S_{ui\neq ij})p(S_{ij}|Z_{1:T}, \Phi, S_{ui\neq ij}) \]

The prior \( p(S_{ij}|S_{ui\neq ij}, \Phi) \) can, in the simplest case, be given by an independent Bernoulli distribution for each case of \( S_{ij} \).