Performance Analysis for Pilot-based 1-bit Channel Estimation with Unknown Quantization Threshold

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Introduction

Considered Problem:
- Channel parameter estimation with a low-complexity receiver performing 1-bit ADC
- Goal: Characterization of performance loss associated with hard-limiting under an unknown threshold
- Approach: Derivation of performance bounds for parameter estimation with nuisance parameter (offset)

System Model - Ideal and 1-bit System

- Receive Signal - Ideal Receiver (∞-bit ADC)
- Transmit Signal (BPSK, Known Structure)
- Receive Signal - Low-Complexity Receiver (1-bit ADC)
- 1-bit ADC - Hard-limiting Model

Deterministic Approach - Hard-limiting Loss

Cramér-Rao Lower Bound - Ideal System with ∞-bit ADC

\[ \text{MSE}_\varphi (\cdot) \triangleq F_{\varphi} \frac{1}{N} \]

Cramér-Rao Lower Bound - Low-Complexity System with 1-bit ADC

\[ \text{MSE}_\varphi (\cdot) \triangleq F_{\varphi} \frac{1}{N} \]

Deterministic - Model Assumptions

\[ \varphi \rightarrow \text{deterministic unknown} \]

Hybrid - Model Assumptions

\[ \varphi \rightarrow \text{random unknown} \]

Deterministic - Estimator (MLE)

\[ \hat{\varphi} (\cdot) \triangleq \arg \max_{\varphi \in \mathbb{C}} p_{\varphi} (\cdot, \varphi) \]

Hybrid - Estimator (MAP / JMAP-MLE)

\[ \hat{\varphi} (\cdot) \triangleq \arg \max_{\varphi \in \mathbb{C}} p_{\varphi} (\cdot, \varphi) \]

Deterministic - Performance (MSE)

\[ \text{MSE}_\varphi (\cdot) \triangleq E_{\varphi} \left[ (\hat{\varphi} (\cdot) - \varphi) \right] \]

Hybrid - Performance (MSE)

\[ \text{MSE}_\varphi (\cdot) \triangleq E_{\varphi} \left[ (\hat{\varphi} (\cdot) - \varphi) \right] \]

Hybrid - Estimator (MAP / JMAP-MLE)

\[ \hat{\varphi} (\cdot) \triangleq \arg \max_{\varphi \in \mathbb{C}} p_{\varphi} (\cdot, \varphi) \]

Expected Cramér-Rao Lower Bound - Ideal System with ∞-bit ADC

\[ \text{MSE}_\varphi \triangleq E_{\varphi} \left[ (\hat{\varphi} (\cdot) - \varphi)^2 \right] \]

Expected Cramér-Rao Lower Bound - Low-Complexity System with 1-bit ADC

\[ \text{MSE}_\varphi \triangleq E_{\varphi} \left[ (\hat{\varphi} (\cdot) - \varphi)^2 \right] \]

Summary and Insights

- Characterization of channel parameter estimation performance under 1-bit ADC with unknown threshold
- Analysis of different estimation approaches (deterministic/hybrid)
- Performance analysis through performance bounds for MLE and MAP/JMAP-ML
- Low SNR regime: additional performance loss (offset estimation) vanishes
- Medium SNR: loss vanishes for small offset values (requires careful 1-bit ADC hardware design)

Further Work

- Analytic characterization of the performance loss in the low SNR regime
- Advanced channel estimation problems (e.g. ISI wireless channel)