Improved Decoding of Analog Modulo Block Codes for Noise Mitigation

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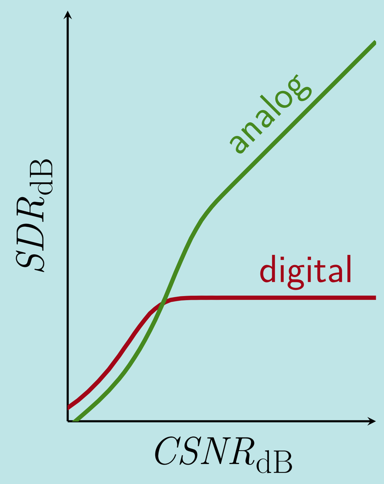
1 Summary

Analog Modulo Block Codes [1] (AMB Codes)

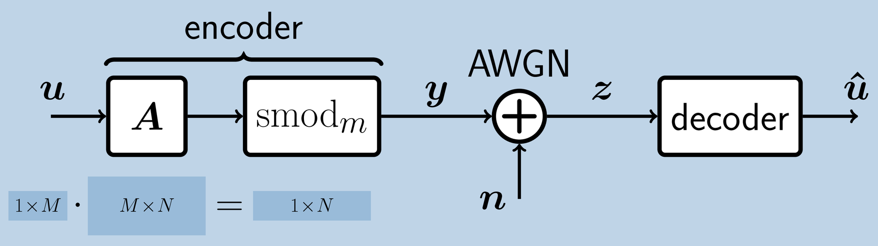
- (Pseudo-)analog channel coding by digital processing
- High resolution ADC (e.g. 16 bits)
  → arbitrary gains for good channels



Improved Decoding

- Fast decoding (Zero Forcing), then ML decoding (if necessary)
  → fast and good decoding results
- Limiting the input signal before (clipping) and after (truncation) decoding
  → further improvement

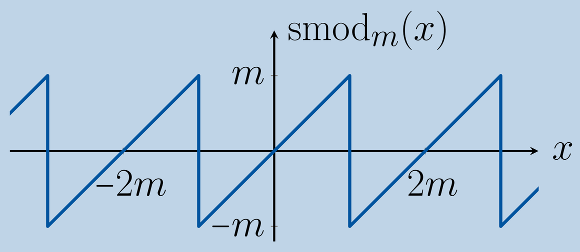
2 AMB Codes | Basics



Encoding

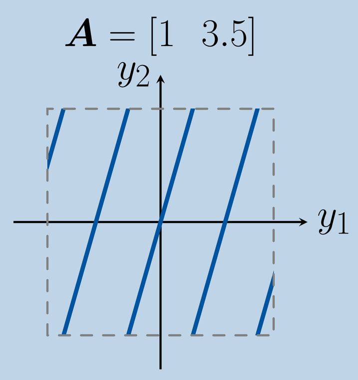
- Inspired by digital block codes:
  Multiplication with generator matrix $\boldsymbol{A}$
- Symmetric modulo function smod to limit transmit power

$$\boldsymbol{y} = \mathrm{smod}_m(\boldsymbol{u}\cdot\boldsymbol{A})$$



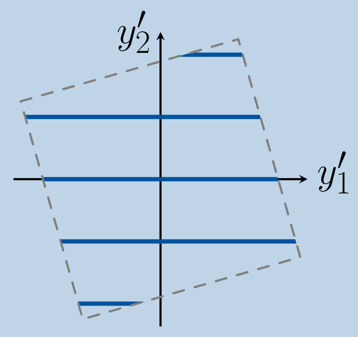
Code Words

- Parallel lines (or hyper-planes)
- Limited to a (hyper-)cube with edge length $2m$ due to the modulo function



Rotation

- Code words $\boldsymbol{y}$ can be rotated, yielding
  - M continuous dimensions $\boldsymbol{y}_{\mathrm{c}} = y_1'$ and
  - $N - M = D$ discrete dimensions $\boldsymbol{y}_{\mathrm{d}} = y_2'$



3 Decoding | State-of-the-art

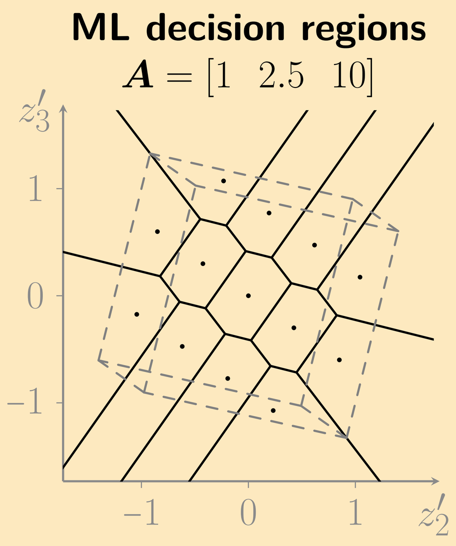
1. Decode discrete dimensions (lattice quantizer) to get estimate $\hat{\boldsymbol{y}}_{\mathrm{d}}$
2. Undo modulo function and decode continuous part
   $\hat{\boldsymbol{u}} = \left(\boldsymbol{z} - \begin{bmatrix}\boldsymbol{0} & 2m\cdot\hat{\boldsymbol{y}}_{\mathrm{d}}\cdot\boldsymbol{B}^{-1}\end{bmatrix}\right)\cdot\boldsymbol{A}^{+}$

Discrete Maximum Likelihood Decoder (DML)

- Determine nearest discrete point

$$\hat{\boldsymbol{y}}_{\mathrm{dDML}} = \arg\min_{\boldsymbol{y}_{\mathrm{d}}} \|\boldsymbol{z}_{\mathrm{d}} - \boldsymbol{y}_{\mathrm{d}}\|$$

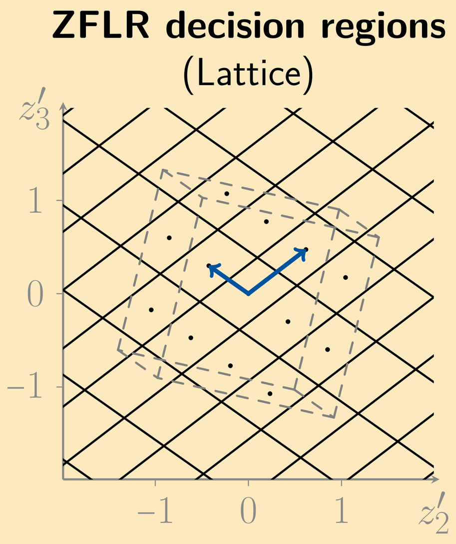
- Many candidate code words $\boldsymbol{y}_{\mathrm{d}}$ (depending on code)
  → *high* decoding complexity



Zero Forcing Decoder (ZFLR)

- Use lattice property for decoding
- Valid code words' discrete part $\boldsymbol{y}_{\mathrm{d}}$ is an *integer* linear combination of base vectors (in $\boldsymbol{L}$)
  → round $\boldsymbol{z}_{\mathrm{d}}\cdot\boldsymbol{L}^{-1}$ to nearest integer:

$$\hat{\boldsymbol{y}}_{\mathrm{dZFLR}} = \lceil \boldsymbol{z}_{\mathrm{d}}\cdot\boldsymbol{L}^{-1} \rfloor\cdot\boldsymbol{L}$$



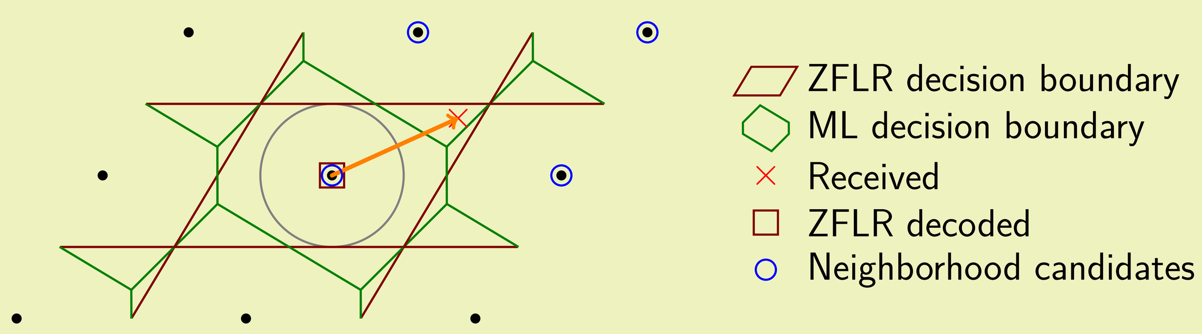
- Decreased quality
- *Low* decoding complexity

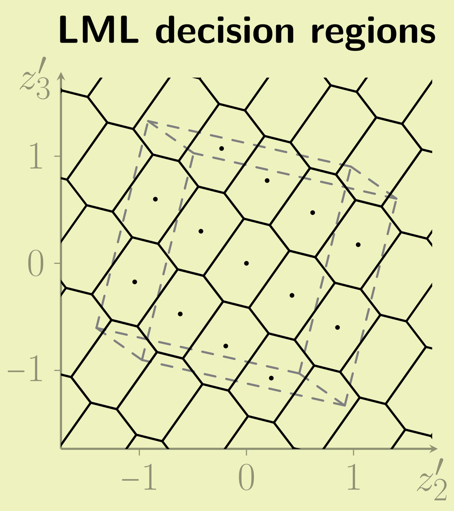
$\boldsymbol{A}^{+}$ Pseudo-inverse of $\boldsymbol{A}$
$\boldsymbol{L}$ Reduced [2] lattice base matrix $\boldsymbol{B}$

4 Lattice ML Decoding (LML)

New!

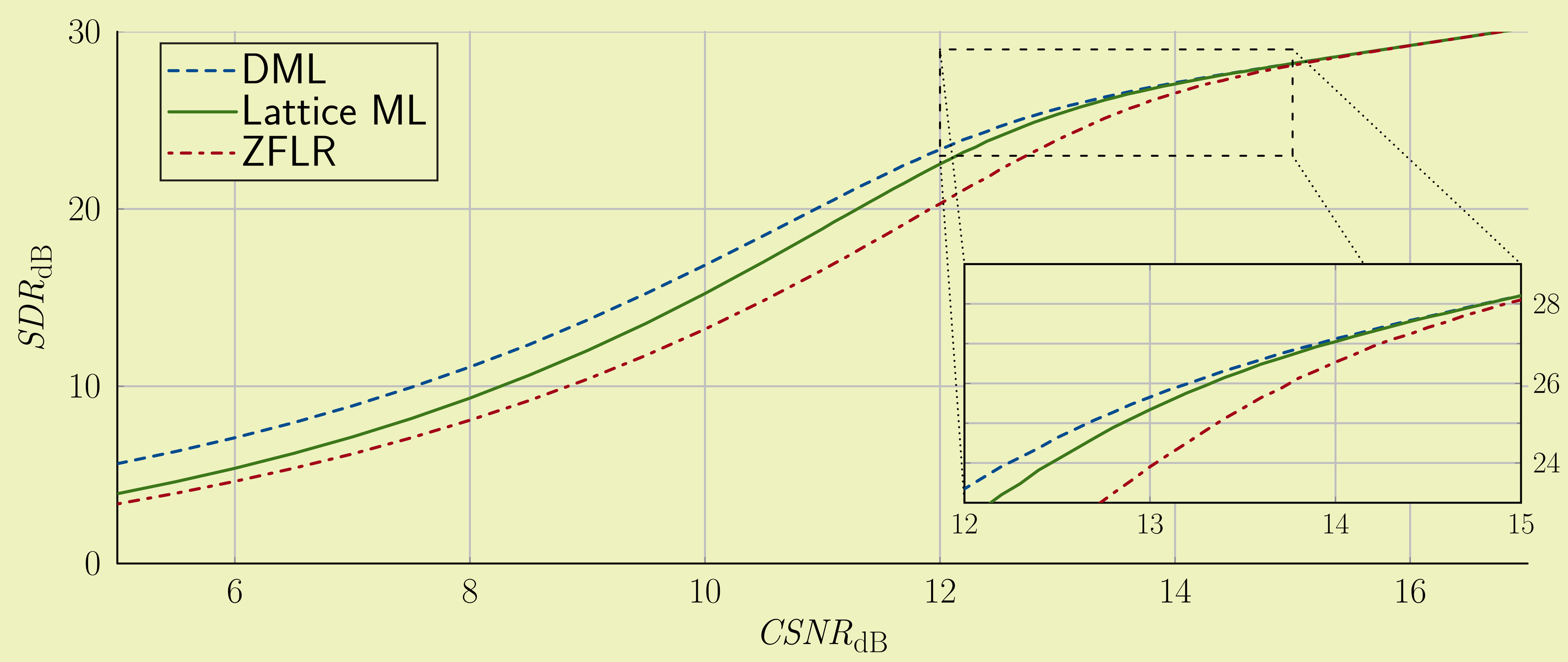
- Combine speed of ZFLR with quality of DML
  Step 1: Zero Forcing Decoding □ ("Lattice", low complexity)
  Step 2: Maximum Likelihood ("ML") search *only* in its neighborhood ○





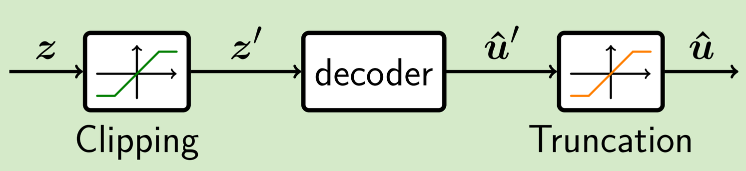
Simulation Results

$\boldsymbol{A} = [1 \;\; 2 \;\; 4]$, uniformly distributed input $\boldsymbol{u}$



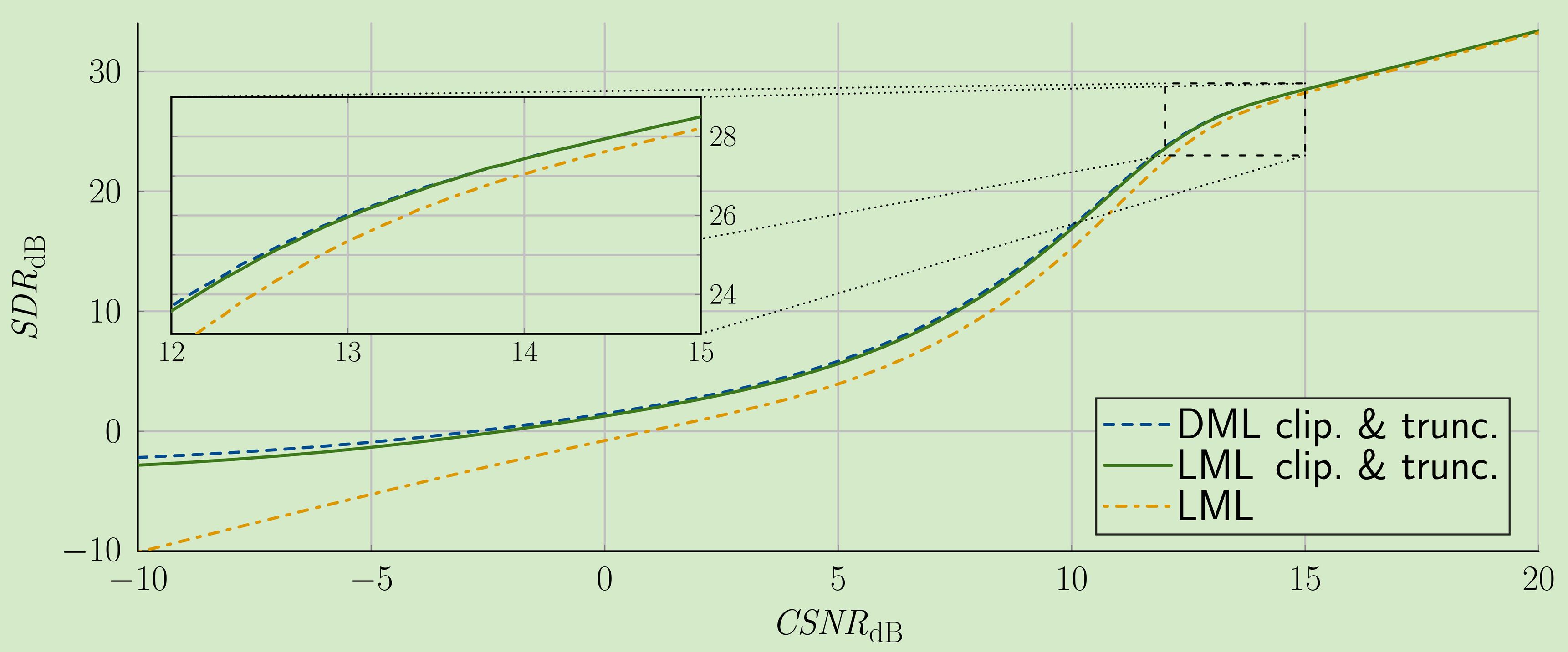
5 Additional Clipping and Truncation

New!



- Limit signal to valid range
  - before decoding ($\pm m$) → clipping
  - after decoding → truncation

Simulation Results



6 Conclusions

LML Decoder

- Significantly reduced complexity (in comparison to DML)

LML decoder with clipping and truncation

- Almost DML performance

References

[1] Tim Schmitz, Matthias Rüngeler, and Peter Vary, "Analysis of analog modulo block codes," in *10th International ITG Conference on Systems, Communications and Coding (SCC'2015)*, Hamburg, Germany, Feb. 2015, available at IEEE Xplore.

[2] Dirk Wübben, Dominik Seethaler, Joakim Jaldén, and Gerald Matz, "Lattice reduction: A survey with applications in wireless communications," *IEEE Signal Processing Magazine*, vol. 28, no. 3, pp. 70–91, May 2011.