Jointly Optimal Power and Rate Allocation for Layered Broadcast Over Amplify-and-Forward Relay Channels

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Motivation

- For single layer transmission, all transmitted bits are equally protected.
- Multi-layer transmission combines
  - Successive refinement layered source coding.
  - Ordered protection levels of the source layers.
- Therefore, the **base** source layer is given higher priority than the **enhancement** source layers.
Motivation (cont’d)

• As a result:
  • For **faded** channel: **Some** information is decoded.
  • For **good** channel: **All** information is decoded.

• Consequently, **outage probability** is decreased.
Motivation (cont’d)

- We are interested in multilayer transmission using broadcast approach:
  - source layers are protected using different channel codewords.
  - All source layers are jointly transmitted using superposition coding.
  - Then they are decoded using successive interference cancellation at the receiver.
- Our contribution is on the investigation of multilayer transmission on a relay channel.
Previous Work

Our Previous Work in terms of multilayer transmission on a relay channel:

- Optimal power allocation for 2-layer transmission over selection relaying decode-and-forward (SDF).
- Optimal power allocation for M-layer transmission over relaying Amplify-and-forward (AF).
Previous Work - SDF Relays

SDF Relays:

- We have investigated the 2-layer transmission.
- $L_1$ is the base layer, and $L_2$ is the enhancement layer.
- $L_2$ refines the description in $L_1$.
- We have applied the SDF strategy.
- We solved the power optimization problem over the 2 layers.
- We found that extending the solution for any number of layers becomes prohibitively complex.
Previous Work - AF Relays

AF Relays:

- We have investigated the Multi-layer transmission for any number of layers \( M \).
- \( L_1 \) is the base layer, and the upper layers are the enhancement layers.
- Each layer refines the information from all lower layers successively (Successive Refinement SR).
- We have applied the AF strategy.
- An approximation was found for the end-to-end channel condition.
- This approximation allows for applying a previous algorithm for solving the power optimization problem.
Preliminaries

Utility function:

- It describes the user satisfaction.
- Function of total rate decoded successfully $\tilde{R}$.
- For example:
  - Maximize expected rate $U(\tilde{R}) = \tilde{R}$
  - Minimize expected distortion $U(\tilde{R}) = 1 - e^{-2\tilde{R}}$
We consider a system that consists of three nodes: source, destination, and relay.

We assume that the source is Gaussian and it is encoded into $M$ layers with fixed rates.

The relay is half-duplex and applies Amplify-and-Forward (AF) strategy.
System Model (cont’d)

- Therefore, the transmission is carried over two consecutive time slots of equal duration and bandwidth.
  - **The first time slot:** The source broadcasts the layers to the relay and the destination.
  - **The second time slot:** If the relay forwards the layers after amplifying to the destination.
• We denote the SNR over the three links of the relay channel using $\gamma_{sr}$, $\gamma_{sd}$ and $\gamma_{rd}$.
• we assume that the source and the relay only know the statistics of the channels.
End-to-End Channel Condition

- The destination combines the two copies from the source and the relay using MRC.
- It was found the end-to-end channel quality $\gamma$ as

$$\gamma = \gamma_{sd} + \frac{\gamma_{sr} \gamma_{rd}}{\gamma_{sr} + \gamma_{rd} + 1}.$$  

- In order to decode a layer $i$, all previous layers should be first decoded successfully.

$$R_j \leq \frac{1}{2} \log \left( 1 + \frac{\alpha_j}{\frac{1}{\gamma} + \sum_{m > j}^{M} \alpha_m} \right) \quad \forall j \leq i.$$  

$$\gamma \geq \bar{\gamma}_i = \begin{cases} \bar{\gamma}_{i-1}, & \frac{1}{2^{2R_i - 1} - \sum_{m > i}^{M} \alpha_m} \\ \frac{\alpha_i}{2^{2R_i - 1} - \sum_{m > i}^{M} \alpha_m} & \end{cases}.$$
Channel Approximation

- The value of $\gamma$ can be bounded as

$$\gamma_{sd} < \gamma \leq \gamma_{sd} + \min(\gamma_{sr}, \gamma_{rd}).$$

- Which can intuitively be written as

$$\gamma \approx \gamma_{sd} + k \min(\gamma_{sr}, \gamma_{rd}),$$

- Therefore for the fading of the channels is Rayleigh distributed, the CDF of $\gamma$:

$$F_\gamma(\gamma) = 1 - \frac{\beta_3}{\beta_3 - \beta'} e^{-\gamma(\beta')} + \frac{\beta'}{\beta_3 - \beta'} e^{-\gamma\beta_3}.$$ 

where $\beta' = \frac{\beta_2 + \beta_3}{k}$, $\beta_1 = \frac{1}{\bar{\gamma}}$, $\beta_2 = \frac{1}{m_1 \bar{\gamma}}$, and $\beta_3 = \frac{1}{m_2 \bar{\gamma}}$. 


The appropriate value for $k$ should be used ($0 < k \leq 1$) such that the approximate CDF becomes as close as possible to the exact CDF of $\gamma$ found numerically.

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Our objective in this paper is to optimally allocate power for the layers in order to maximize the expected utility function:

$$\max_{\alpha, R} \int_{0}^{\infty} f_\gamma(\gamma) \ U(\bar{R}(\gamma, \alpha, R)) \, d\gamma$$

subject to

$$\sum_{i=1}^{M} \alpha_i = 1, \quad \alpha_i \geq 0 \quad \forall i,$$

We start by doing the change of variables step:

$$b_i = 2^{2R_i} - 1,$$

$$\bar{n}_i = \frac{1}{\bar{\gamma}_i}.$$
Then we have the following problem

$$\max_{b, \tilde{n}} \sum_{i=1}^{M} U(b_i) \left( F_n(\tilde{n}_i) - F_n(\tilde{n}_{i+1}) \right)$$

subject to

$$\sum_{i=1}^{M} b_i (\tilde{n}_i - \tilde{n}_{i+1}) = 1,$$

$$0 < \tilde{n}_M \leq \tilde{n}_{M-1} \leq \ldots \leq \tilde{n}_1,$$

$$b_M \geq b_{M-1} \geq \ldots \geq b_1 > 0,$$

where $c_i = U_i - U_{i-1}$. 
Problem Formulation

- It was found by [Shaqfeh '13] that a unique solution exists for this problem with using the full number of layers.
- Also, a strong duality between the primal and the dual problem is guaranteed.
- Then we have the following KKT conditions ($2M + 1$ equations):

$$\frac{\Delta U_i}{\Delta b_i} f_n(\bar{n}_i) = \lambda \quad \forall i,$$

$$\frac{\Delta F_i}{\Delta \bar{n}_i} U'(b_i) = \lambda \quad \forall i,$$

$$\sum_{i=1}^{M} b_i(\bar{n}_i - \bar{n}_{i+1}) = 1,$$
Problem Formulation

- We can solve this problem by doing 2-dimensional bisection search over $\lambda$ and $n_M$ to find $n_i'$s and $b_i'$s.

- Hence we can find $\gamma_i'$s and $R_i'$s (Optimal rates and channel thresholds).
Maximize Expected Rate

Objective: Maximize Expected Rate

Expected Rate (bits/sec/Hz)

Average $\gamma_{sd}$ in dB

- Infinite Number of Layers
- Four Layers–Optimal Allocation
- Three Layers–Optimal Allocation
- Two Layers–Optimal Allocation
- One Layer–Optimal Allocation
Maximize Expected Rate

- Infinite Number of Layers - Upper Bound
- Four Layers - Optimal Rate and Power
- Four Layers - Unoptimal Rate, Optimal Power
- Four Layers - Unoptimal Rate and Power

Expected Rate (bits/sec/Hz) vs. Average $\gamma_{sd}$ in dB

0 5 10 15 20 25 30 35 40
Minimize Expected Distortion

![Graph showing Minimize Expected Distortion](image_url)

- Infinite Layers – no relay
- Infinite Layers – AF relay – (m1,m2)=(16,16)
- Infinite Layers – AF relay – (m1,m2)=(8,4)
- Infinite Layers – AF relay – (m1,m2)=(100,1)
Optimal Rates for M=3

![Graph of Rates For Layers (bits/sec/Hz)](image)

- Layer 1 – Max Rate
- Layer 2 – Max Rate
- Layer 3 – Max Rate
- Layer 1 – Min Distortion
- Layer 2 – Min Distortion
- Layer 3 – Min Distortion

Average $\gamma_{sd}$ in dB

Rates For Layers (bits/sec/Hz)
Optimal Power Ratios for M=3

![Graph showing optimal power ratios for different layers. The x-axis represents the average $\gamma_{sd}$ in dB, ranging from 0 to 40. The y-axis represents power ratios for layers, ranging from 0 to 1. The graph includes lines and markers indicating different layers and their optimal power ratios.]
Conclusions

- We have considered Multilayer transmission with M layers using the broadcast approach.
- A relay has been considered that applies AF strategy.
- We have proposed a simple approximation for the end-to-end channel statistics.
- We found a unique solution of using the full number of layers.
- We have shown that with a relatively small number of layers, we can approach the upper bound corresponding the infinite number of layers case.
- The numerical results demonstrate that for high values of SNR, the no-relay case may show better performance.
Thank you for your time and attention. Questions?