Information Point Set Registration for Shape Recognition
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Introduction
- This paper proposes a way of enhancing shape recognition through point set registration.
- Point correspondences between two shapes are obtained by the flip invariant shape context. Then, the query shape \( Y \) is registered to the template shape \( X \) using information theoretical learning (ITL) techniques.

Flip Invariant Shape Context
- Shape context (SC) is a well-known descriptor for point sets. Any 2-D point is described by a histogram \( h \) binned in distances and angles relative to other points.
- Point correspondences are then found by calculating the matching cost of any two points \( x_i \) and \( y_j \):

\[
C(x_i, y_j) = \sum_{k=1}^{N} \frac{(h_k(x_i) - h_k(y_j))^2}{h_k(x_i) + h_k(y_j)}
\]

(1)

and minimizing the overall cost using Hungarian algorithm.
- SC suffers from not being flip invariant, as well as the ambiguity of the direction of the tangent line. This renders a total of 4 conditions for SC.

Affine and Non-rigid Transformation
- With correspondences available, affine registration becomes a well-defined optimization problem: for point set \( X = \{x_i\}_{i=1}^{N} \) and \( Y = \{y_i\}_{i=1}^{N} \), find the transformation matrix \( A \):

\[
A = \text{argmax} \sum_{i=1}^{N} G_c(x_iA, y_i)
\]

(2)

where \( G_c(x, y) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(x-y)^2}{2\sigma^2}\right) \). This is called the maximum correntropy criterion (MCC), which is more robust to outliers than MSE.
- The fixed point solution to (2) is

\[
D = \text{diag}(G_0(f(x_1), y_1), ..., G_0(f(x_N), y_N))
A_{corr} = (f(X)^T D(X)^{-1}) (f(X) D(Y))
A = A_{corr} A_{affine}
\]

(3)

Convergence is guaranteed and stopping criterion can be easily set.
- Certain amount of non-rigid transformation is helpful for overcoming intra-class deformation. The Cauchy-Schwarz divergence \( D_{CS} \) describes two PDFs' similarity. A regularized \( D_{CS} \)-based cost function can be written as

\[
J = -2\log \sum_{i=1}^{N} \sum_{j=1}^{N} G_c(y_j, t_i +KW) + \log \sum_{i=1}^{N} \sum_{j=1}^{N} G_c(t_iKW, t_i + t_jKW) + \lambda \text{str}(W)\text{KW}
\]

(4)

where \( T = XA, K \) is the TPS matrix and \( W \) is the transformation matrix to be determined using fixed point solution in a manner similar to (3).

Shape Similarity Criterion
- A correntropy based shape similarity measure is:

\[
\text{corr. cost}(X, Y) = \sum_{i=1}^{N} G_c(y_i, \text{fournigrid}(f_{affine}(x_i)))
\]

(5)

This measure can be combined with the conventional SC-based similarity measure:

\[
\text{new cost}(X, Y) = \frac{\text{corr. cost}(X, Y)}{SC \cdot \text{corr}(X, Y)}
\]

(6)

The correntropy cost is able to suppress bad SC matches such that their effects are nearly negligible.

Experimental Results
- Shape registration: the query shape is registered to the template shape.
- Shape retrieval (Kimia-99 dataset): shown in the table are the "bull's eye" score.
- Marine animal classification: 5 instances shown are "templates". A "query" is classified as the specie of the template that produces highest matching score.

<table>
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<th>Method</th>
<th>Island</th>
<th>Proteus</th>
<th>Penguin</th>
<th>Turtle</th>
<th>Total</th>
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<td>97</td>
<td>97</td>
<td>95</td>
<td>95</td>
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<tr>
<td>Corr. (affine only)</td>
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<td>98</td>
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<td>SC+Corr.</td>
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<tr>
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</table>

- Both registration and recognition results outperform or matches up with state-of-the-art algorithms.